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Paper:

Force Compensating Trajectories for Redundant Robots: Experimental Results

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We proposed a new approach for redundant robots trajectories planning, based on the Null space (or Kernel) features. The Null space (Kernel) exists only in the case of redundant robots and it describes these joints motion which do not affect the robot end-effector motion in the sense of both position and orientation. Based on this "hidden motion" realized in the configuration space, which does not affect the motion in the working zone, we can control independently the robot end-effector position and orientation motions, or just maintain its state while some external force is applied to it.

The proposed control strategy is simple, no additional penalty functions are used to restraint the end-effector motion as in the case of the conventional methods. No pseudo inverse kinematics calculations are required; the desired trajectories are generated directly in the configuration space. No complicated control schemes are introduced, the proposed method is based on solving algebraic systems of equations and finding eigenvectors and eigenvalues.

In the paper the results from simulations and experiments based on the proposed method are presented and discussed.

Keywords: redundant robots, trajectories planning, external force compensation, kernel

1. Introduction

Redundant robots have been studied for many years, using mainly their features for obstacle avoidance [1, 3–5], creating algorithms based on some energy function minimization or other optimization criteria [6, 7]. Basic mathematical tools are pseudo-inverse Jacobian and the control strategies have been developed based on their kinematics mainly [2, 8–9].

This study has been inspired by the human hand features and our desire to translate them in the robots world. The human hand can approach a given object in various ways and apply different forces to the environment. It has 7 degrees of freedom excluding the fingers, so in this work a 7 d.o.f. robot with revolute joints has been investigated.

In the case of redundant robots we do not have one to one correspondence between the robot configuration space Q and working space [10, 11]. There are some joint motions which do not affect the robot end-effector motion. In the next, these joint trajectories will be called Kernel trajectories. Further such trajectories are generated and it is analyzed how to use them in order to maintain the robot end-effector state while some external force is applied to the last link.

The kernel trajectories practically assure redistribution of the inner passive forces in the redundant system, keeping the established state in the working zone. The conventional control methods, aiming mainly at obstacle avoidance are realized by fixation of some of the joints and controlling the remaining ones, based on some additional optimization criteria. Here the joints to control are derived directly from the Kernel trajectories. The originality of the method consists of the use of some non traditional mathematical tools as homomorphisms, groups and factor-groups, manifolds, orthogonal fibrations. That leads to quite different point of view exceeding the traditional considerations, for example decoupling of the configuration space into 4 different subsets of vectors, each of them keeping features for different type of control.

In [12] how the configuration space can be partitioned into 4 independent subsets of vectors, each corresponding to a different end-effector motion in the working space, has been described. In [16] only one of the above mentioned subsets of configuration vectors has been considered in details, i.e. the one responsible for positioning the endeffector without changing its orientation. Here, the subset of configuration vectors generating no motion of the robot end-effector (maintaining the state unchanged)

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has been described in details and its practical application illustrated.

2. Theoretical Background

In order to describe the transformation between the robot joint space and working zone the sensibility theory has been applied. Theoretically the sensibility is defined as a homomorphism transforming a set of vectors from the configuration space (approximated by a smooth surface as its border – ball with radius ε) into a sensibility ellipsoid in the working zone [12]. The last contains the errors from the theoretical position and orientation of the end-effector, caused by some deviations of the joint variables δq [12]. The homomorphisms consists of two parts τ_p and τ_r for position and orientation.

In the case of position, the deviations of the end-effector δR from the desired state, caused by some deviations of the joints δq (due to different errors sources) are given by:

$$\delta R = A\delta q \quad \text{. . . . . (1)}$$

where the elements of the matrix A are $a_{ij} = \frac{\partial R_i}{\partial q_j}$, $i = 1,\ldots,3; \quad j = 1,\ldots,n$ (n – number of joints). The matrix A is the matrix of the linear operator of the homomorphism τ_p for position. Analogously, in the case of orientation the deviations from the desired system state are described by the vector of small rotations:

$$\delta\theta = L\delta q. \quad \text{. . . . . (2)}$$

Where $l_{ij} = \frac{\partial \theta_i}{\partial q_j}, i = 1,\ldots,3; j = 1,\ldots,n$, are the elements of the matrix L, which is the linear homomorphism operator in the case of orientation. Having kinematically redundant robot, part of the vectors form the configuration space is transformed in the zero vector in the working space, by the transformations (1) and (2). These vectors belong to the null space or Kernel of the homomorphism and do not affect the end-effector state.

To determine the relation between both homomorphisms τ_p and τ_r, the ε-sphere and the sensibility ellipsoid are considered as groups with vector elements. For every homomorphism of the group ε into the group E_ε^H (i.e. the sensibility ellipsoid) the set of elements, transformed into the neutral element (zero vector) is a normal divider G_E (i.e. $\mathrm{Ker}\,\tau_p$) of the group ε. The set of elements, transformed into arbitrary non-zero elements in the group E_ε^H form some neighbor class with respect to the kernel G_E. The factor-group ε/G_E is isomorphic to the group E_ε^H. Or, each normal divider G_E of the group ε is the kernel of a homomorphism, transforming ε into the factor-group ε/G_E. This approach, puts no restrictions on any axes, kinematics structure (three-like or not) neither on the type of joints. That can be pointed out as an advantage of the method. Such analysis is easier to be done using the isomorphism theorem between the working zone and factor-groups of homomorphism kernels as normal dividers. It is well known that the kernel of any homomorphism between two groups is in the same time normal divider of one of them. We can consider two homomorphisms between a ball round theoretical state and sensibility ellipsoids for position and orientation. It is also known that the intersection of any normal dividers themselves is also a normal divider of the group. On the other hand, the considered group can be factorised with respect to every normal divider. In our case two factor-groups can be built for position and orientation state separately. In the same way one more factor-group can be constructed having the intersection of both homomorphisms mentioned above as a normal divider. By definition it is called sensibility factor-group. And its elements have very clear mechanical interpretation. Part of them looks after the position only, another part – the orientation only and the remaining – for both of them. The unit element itself (the vectors of that neighbor class) will not affect the theoretical state. This way, we can make a common description of position and orientation motions which allows dividing the configuration space into four orthogonal subsets (fibrations) with vector elements [12]. The last gives the possibility to use these sets of vectors for generating desired joint trajectories, allowing controlling independently position and orientation motions. Or, if we use the vectors describing both homomorphisms Kernel intersection, we can maintain the end-effector state, while some external force is applied to it. This approach is applicable only for redundant robots, because only then we have Null space or homomorphisms Kernel intersection.

Further the following notations are introduced [12]. The set of the vectors belonging to the images intersection of both homomorphisms, $\{\mathrm{Im}\,\tau_p \cap \mathrm{Im}\,\tau_r\}$ is denoted by $SF_{pr}(n)$, (n – number of joints). The set of the vectors of both Kernels intersection $\{\mathrm{Ker}\,\tau_p \cap \mathrm{Ker}\,\tau_r\}$, is denoted by $SF_0(n)$. The set of vectors from the image of orientation and the Kernel of position $\{\mathrm{Ker}\,\tau_p \cap \mathrm{Im}\,\tau_r\}$, by $SF_{r0}(n)$. The set of vectors from the image of position and kernel of orientation $\{\mathrm{Im}\,\tau_p \cap \mathrm{Ker}\,\tau_r\}$, by $SF_{p0}(n)$. Every fibration contains infinite number of vectors. But there exist finite number of independent vectors there. They are eigenvectors for the tasks $(A^TA - \lambda E)X = 0$ and $(L^TL - \lambda E)X = 0$, where E is the unit matrix of order n. Such a way some basis is defined (pre-images of sensibility directions or the eigenvectors corresponding to zero eigenvalues) all the vectors could be expressed by. They are called basic neighbor classes of the orthogonal fibrations [12].

3. Kernel Trajectories Generation

In the next, in order to make the robot resist to some external influence (force – acting on the last link), by maintaining its end-effector state unchanged, we need to generate such joint trajectories, which belong to both kernels (for position and orientation) intersection $\{\mathrm{Ker}\,\tau_p \cap \mathrm{Ker}\,\tau_r\}$. In other words we need to generate such joint motions which do not affect the end-effector motion.

Further analysis and experiments are accomplished for the trajectories belonging to the set $SF_0(n)$. All possible trajectories, which the system could realize in $SF_0(n)$ will be called *Kernel trajectories*. Belonging to $SF_0(n)$, the vectors $q(t)$ would not affect the system orientation and position but would allow the system to react to some external force. They could be obtained by solving the system (where A and L have been defined in the previous section) [13]:

$$\left| \begin{array}{l} Aq = 0 \\ Lq = 0 \end{array} \right. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

which has six equations and n variables, i.e. as $n > 6$, the system *always* has fundamental solutions based on $(n-6)$ independent parameters [14].

In (3) A and L are derived for a fixed configuration (a point of the robot configuration space). Such a configuration point could be understood as a "description" of both homomorphisms. The center of the ε-ball is calculated for it. Local coordinates can be introduced by that part of the eigenvectors, which form the basis of the Kernel. The considered Kernel trajectory is described by local coordinates and belongs to corresponding ε-ball (given configuration). It is normal to expect that the control of the robot along that part of such a Kernel trajectory would be worse when the robot moves far from the ball center. As it will be seen later, the experiments confirm the same. Then the Kernel trajectory must be prolonged using the manifold structure induced over the robot configuration space [12]. The configuration space Q can be considered as a subset of linear space with vectors $q = (q_s)_{s=1}^{n}$, which has borders because of the physical or constructive reasons. Let by q_p an arbitrary vector is denoted, and by Q_ε^p – a set of vectors q_s^ε, which difference from q_p is smaller by length than a preliminarily given number ε, i.e. $Q_\varepsilon^p = \left\{ q_s^\varepsilon : \left| q_s^\varepsilon - q_p \right| < \varepsilon \right\}$. This set is n-dimensional sphere with center "the point" q_p and radiusε. If we introduce the notation $\delta q_s^\varepsilon = q_s^\varepsilon - q_p$, then we can write Q_ε^p in the form $Q_\varepsilon^p = \{\delta q_s^\varepsilon : |\delta q_s^\varepsilon| < \varepsilon\}$. That way the configuration space Q can be presented as a union of enumerated areas Q_ε^p round every point of the set Q. If the point q_p is considered as "a theoretical robot state", the area Q_ε^p is interpreted like a pre-image of the homomorphism which transforms it into the sensibility ellipsoid. Also an orthonormal basis of eigenvectors exists which image represents the kinematics sensibility directions corresponding to q_p. From (1) we have:

$$\delta R^2 = (\delta R)^T (\delta R) = (A\delta q)^T (A\delta q) = \delta q^T A^T A \delta q = \delta q^T B \delta q.$$

The matrix B is symmetric and positive defined one. So, for such matrices orthonormal basis of eigenvectors exists. The same can be said for any other point q_d of Q. Then its area will be Q_ε^d. The orthonormal basis of eigenvectors mentioned above allows introducing the local coordinates $(x_p^\alpha)_{\alpha=1}^n$ in the area Q_ε^p. It can be done also for Q_ε^d where the corresponding local coordinates are $(x_d^\alpha)_{\alpha=1}^n$. Then in the intersection $Q_\varepsilon^p \cap Q_\varepsilon^d$ both kinds of local coordinates will be valid. From the theory it is known that for every two orthonormal bases of one and the same linear space (the subsets Q_ε^p belong to the configuration space Q) there exists regular transformation between them. That means every system of local coordinates can be expressed by another one and vice versa:

$$x_p^\alpha = x_p^\alpha (x_d^1, \ldots, x_d^n), \quad \alpha = 1, \ldots, n$$

$$x_d^\alpha = x_d^\alpha (x_p^1, \ldots, x_p^n), \quad \alpha = 1, \ldots, n$$

and the Jacobian det $\left(\frac{\partial x_p^\alpha}{\partial x_d^\alpha} \right)$ is different from zero. In this way a manifold structure over robot configuration space can be introduced. The ε-balls form the union, which covers the configuration space. The homomorphisms eigenvectors, which are a basis of the Kernel, introduce local coordinates round the ball center. The transformation between the local coordinates is assured by the connection of any pair of orthonormal basis vectors in the linear vector space, which is expressed by the corresponding orthogonal matrix. As a result, the new homomorphisms are defined, derived by new configuration points and the Kernel trajectory is formulated in the terms of the new local coordinates and so on. That theoretical case is realized practically by "updating" many times the matrices A and L with the current joint values during the control of the robot along the Kernel trajectory.

The system (3) represents a mapping between the configuration space and working zone and its right side equal to zero is equivalent to joint motions which do not cause an end-effector motion.

In our case a 7 d.o.f. robot has been controlled, which has only rotational joints. It has three dimensional Images and four dimensional Kernels in the case of position and orientation respectively. Further only the $SF_0(n)$ features will be discussed. For the considered robot, the fibration $SF_0(n)$ is in the general case one-dimensional (the robot has 7 d.o.f.; solving the system (3) and having 6 equations we obtain the fundamental solution based on one independent parameter). For some configurations the $SF_0(n)$ dimension could increase giving better possibility for force interaction with the environment. Having a robot with greater number of joints, the number of independent parameters (solutions) increases and we have a variety of kernel trajectories we can generate.

4. Control Signal Synthesis

The control signal has been synthesized based on a modified computed torque method, with additional corrections for the joint positions and velocities. It is presented in details in [15]. Here we will introduce it briefly. In the general case the robot-manipulator's dynamic is described by:

$$A(q, d)\ddot{q} + h(q, \dot{q}, d) + g(q, d) + N(q, F_{ext}) = \tau, \quad (4)$$

with q the joint-variable vector, τ the generalized force/torque vector, $A(q, d)$ the inertia matrix, $h(q, \dot{q}, d)$ the Coriolis/centripetal vector, $g(q, d)$ gravity vector,

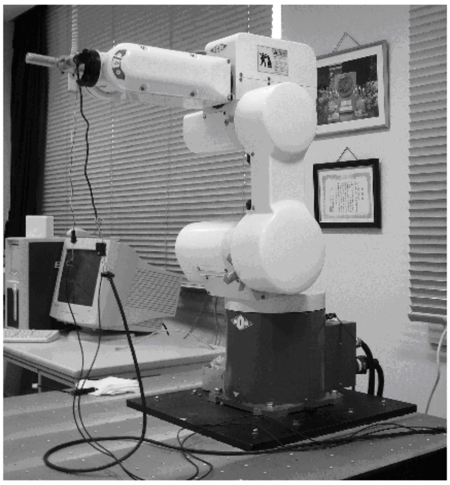

Fig. 1. The controlled robot.

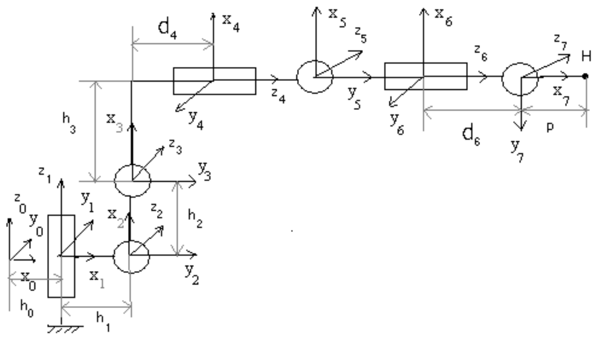

Fig. 2. Kinematics scheme of the robot.

$N(q, F_{ext})$ external force components, translated in the joint space and d the robot geometric parameters. In more compact form, (4) could be written as:

$$A(q,d)\ddot{q}+B(q,\dot{q},d,F_{ext})=\tau \quad (5)$$

The arm control input in its general form is [15]:

$$\tau = A\ddot{q}+B-K_2(\Delta\dot{q}-\dot{z})-K_1(\Delta q-z)-(\Delta\ddot{q}-\ddot{z}) \quad (6)$$

where K_1 and K_2 are diagonal feedback coefficients matrices with positive elements; z, $\dot{z}$, $\ddot{z}$ are additional corrections for the joints positions, velocities and accelerations, derived by solving the system (7):

$$\ddot{z}(t)+K_2\dot{z}(t)+K_1z(t)=KV(t), \quad t\in[t_{i-1},t_i] \quad (7)$$

where K is constant matrix and its elements have weighty coefficients meaning, $V(t_i)=M_1V_1(t_i)+M_2V_2(t_i)+M_3V_3(t_i), V_1(t)=q(t)-q_d(t)=\Delta q, V_2(t)=\dot{q}(t)-\dot{q}_d(t)=\Delta\dot{q}, V_3(t)=\ddot{q}(t)-\ddot{q}_d(t)=\Delta\ddot{q}$; M_1, M_2, M_3 – dimensional coefficients; $q_d, \dot{q}_d, \ddot{q}_d$– desired joints positions, velocities, accelerations. We have neglected the Coriolis forces and moreover the desired joint trajectories are linear, i.e. the acceleration is zero. Therefore the overall robot arm input in our particular case has the form:

$$\tau = g(q,d)+N(q,\ F_{ext})-K_2(\Delta\dot{q}-\dot{z})-K_1(\Delta q-z). \quad (8)$$

In (8) the external force F_{ext} components are measured by a six-component force/torque sensor and could be translated into a joint torque by using the transposed Jacobian.

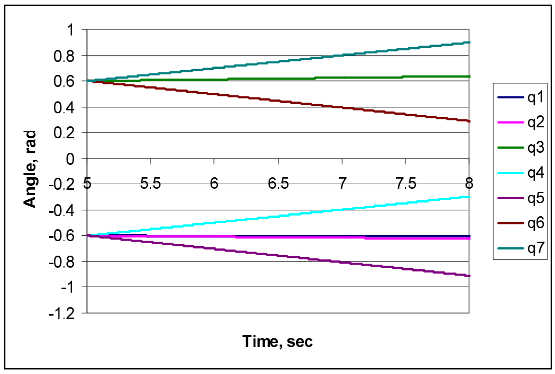

Fig. 3. Desired kernel trajectories for the joints (1 Kernel traj.).

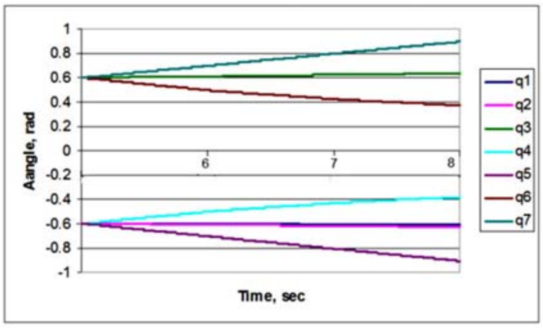

Fig. 4. Desired kernel trajectories for the joints (3 Kernel traj.).

5. Experimental Results

As it was mentioned above the human hand has 7 degrees of freedom, excluding the fingers, that is why we applied our control scheme to a 7 d.o.f. robot with only revolute joints. It is shown on **Fig. 1**. Its kinematics scheme is shown on **Fig. 2**.

This robot arm (VS6354B, made by DENSO company), has 6 d.o.f. and one additional joint attached in order to apply the proposed control method, which is valid only in the case of redundant robots. Totally we have 7 d.o.f. robot arm. Its geometrical dimensions are $h_1 = 0.1$ m, $h_2 = 0.255$ m, $h_3 = 0.085$ m, $d_4 = 0.285$ m, $d_6 = 0.01$ m, $p = 0.1$ m.

The objectif of the experiments is to apply some external force to the robot end-effector and apply at the same time some desired joints Kernel trajectories which maintain the robot end-effector unchanged although the applied external force. In that sense we could say these trajectories will compensate the applied external force. A 6 component force-torque sensor has been mounted between the 6th and 7th body in order to measure the applied external force. To facilitate the realization of the experiments, the desired joints Kernel trajectories have been generated in advance. For better results and more precise control they should be generated in real time. In order to decide the desired joint trajectories, some simulations have been done. First the system (3) has been solved, taking the values of the joint angles at the initial contact (at the moment when starting applying some force) and without updating the system (3) with the current joints val-

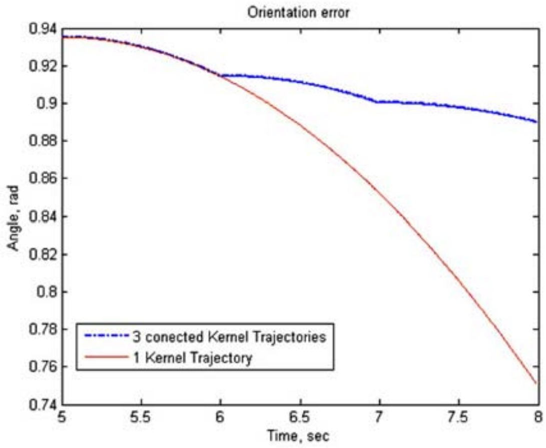

Fig. 5. Theoretical orientation trajectory without external force.

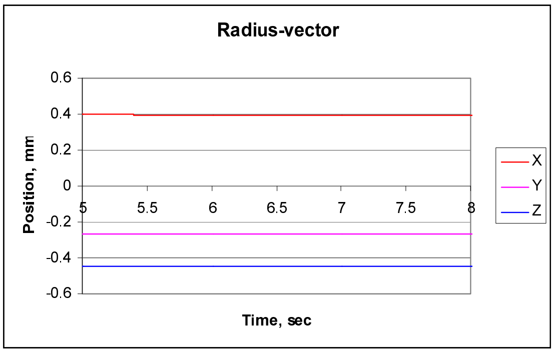

Fig. 6. Theoretical position error without external force (1 Kernel traj).

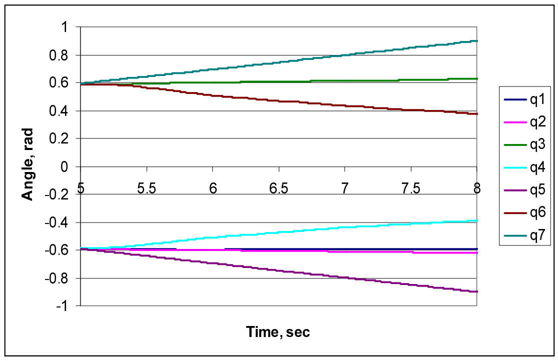

Fig. 7. Measured joints kernel trajectories.

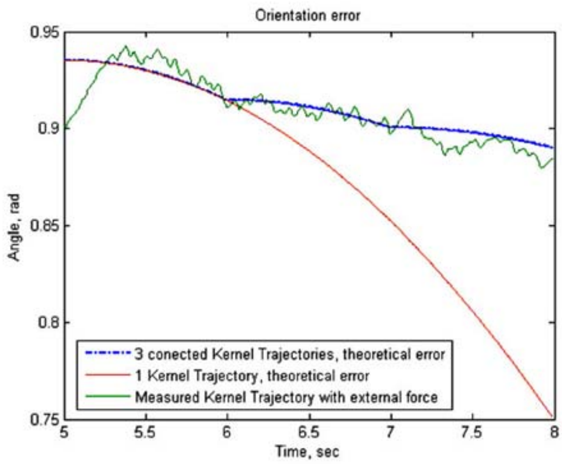

Fig. 8. Measured and theoretical orientation trajectory.

ues during the control. The later means we do not switch between different Kernel trajectories (we do not have a transition between different ε-balls in the configuration space). That is why here the deviation for the end-effector position and orientation respectively is large. The generated joints Kernel trajectories are shown on **Fig. 3**. The orientation error is presented on **Fig. 5** and it is 0.1846 rad. Position error (**Fig. 6**) is $[0.007263, 0.00443, 0.00421]$ m. Therefore this trajectory is not efficient for our control purposes.

Other Kernel trajectories have been generated, this time solving the system (3), updating it every 1 sec with the current joints values. The total motion is 3 sec, therefore we have connected 3 Kernel trajectories (transition between 3 ε-balls in the configuration space). The desired joints trajectories are shown on **Fig. 4**. Here the end-effector state has been maintained. The orientation error is smaller – 0.0454 rad. It is shown on **Fig. 5**. From the results it can be seen that the orientation of the end-effector is not kept when we do not switch between several joint trajectories, however in the case we update more often the system (3), the orientation error is smaller. For describing the orientation we have used Euler parameters, describing a finite rotation about an arbitrary axis. They are an alternative to Euler angles, but the advantage of using them is, they allow a singularity-free description of rotational motion and give equivalent result concerning orientation errors estimation.

For the considered robot we have only one independent parameter in the system (3) and it is an algebraic system of 6 equations. For both cases we have decided linear law of variation of the independent parameter q_7 . By choosing the independent parameter law of variation we can control the kernel trajectory direction or amplitude. However, if some higher magnitude force is applied to the end-effector, and physically the robot is not capable to maintain it, then it will be driven to a singular configuration by the Kernel trajectories. The force magnitude and direction, which could be compensated, depend on each concrete robot kinematics scheme and motors power. Further, the 3 connected Kernel trajectories presented on **Fig. 4** have been chosen as desired ones for the real control. For one of the experiments a desired Kernel trajectory without been updated has been tracked for comparison. Next, by applying different forces to the last robot link, pushing it from different sides, we have investigated how this influences the robot end-effector state. In the first experiment we have pushed the 7^{th} link frontally. The results are presented on **Fig. 7** – measured joint Kernel trajectories; **Fig. 8**– measured and theoretical orientation errors; **Fig. 9**– measured end-effector position; **Fig. 10** – measured external forces.

The measured force is presented with respect to the sen-

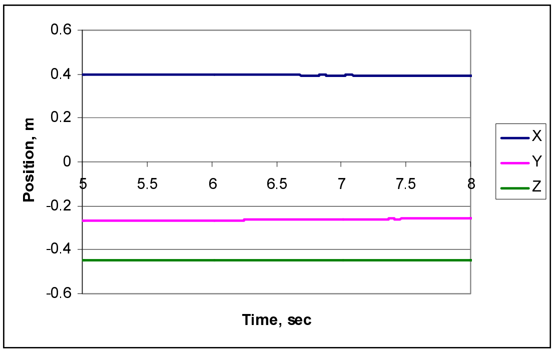

Fig. 9. Measured end-effector position.

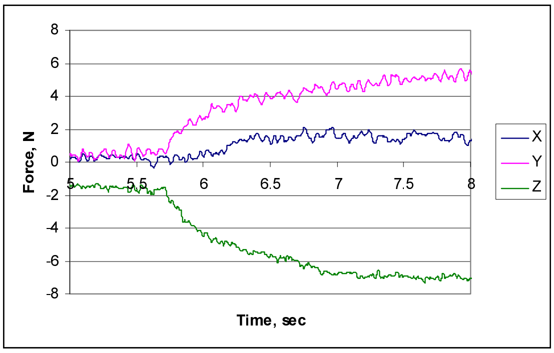

Fig. 10. Measured external force.

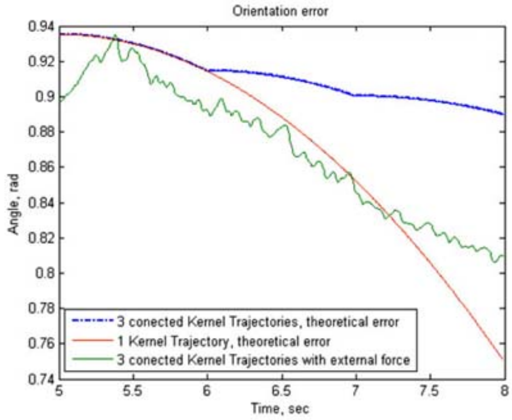

Fig. 11. Measured and theoretical orientation trajectory.

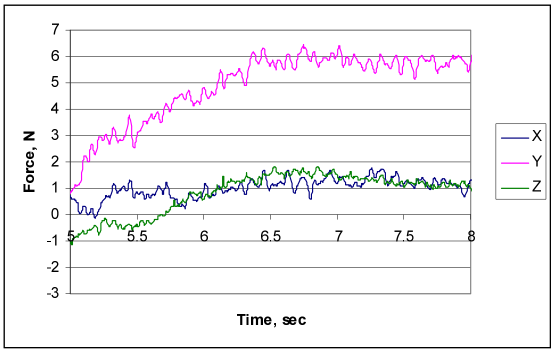

Fig. 12. Measured external force.

sor coordinate frame. As it is seen from the results the orientation of the end-effector has been maintained along the trajectories, compared with the theoretical evaluation about the orientation error. The deviation is 0.0171 rad. Concerning its position, along the X and Z directions we have small deviations about 1 or 2 mm, but along Y axis we have bigger deviations, about 4 mm difference from the theoretically estimated error. If we take a closer look at the measured Kernel trajectories (**Fig. 7**) and desired ones (**Fig. 4**) we can notice that the robot joints have been changing dependently in pairs. For example q_2 and q_3 are changing in the opposite direction, q_4 and q_6 are changing dependently and q_5 and q_7 are changing dependently. The first join q_1 is not changing. If we consider the robot kinematics scheme (**Fig. 2**) we can see that the joints are grouped by pairs and each joint is responsible for compensating some other joint deviation, which could be caused by some external disturbance. Only the first joint does not have an analogous one moving along the same axis that is why the first joint trajectory is almost zero. These trajectories come directly from the solution of the system (3). We can expect that for this concrete robot, if some external disturbance provokes some deviation of the first joint then it could not be compensated by the Kernel trajectories. In this experiment we have generated the Kernel trajectory in advance, by taking into account the theoretical values of the joints. But practically this leads to some deviations from the robot end-effector state. The Kernel trajectories have to be computed in real time, by updating the matrices A and L from (3) with the measured joints values. Each updates of A and L is a beginning of new Kernel trajectory, which means, we need to generate several short kernel trajectories and switch between them during the real control. This way, if some joint position has been changed by the external force (for ex. 6^{th} joint), the corresponding pair joint motion - (4^{th} joint), will be generated in real time in order to compensate the deviation. Also, the computed kernel trajectory could be updated more often. In our case the interval for updating it is still large – 1 sec, practically it could be shorter, for instance every 3 or 4 steps. In the next we have applied some force on the last link from bellow and we had not updated the desired trajectory during the experiment. On the **Fig. 11** and **Fig. 12** we have presented the results (end-effector orientation errors, measured and theoretically estimated; and measured force).

As it is seen from the measured orientation error of the end-effector (**Fig. 11**), the end-effector orientation could not be maintained in that case. The error is 0.087 rad. We have observed also big errors on the end-effector position reaching 20 mm in Z direction and 11 mm in Y direction. As it is seen from the measured joint trajectories (**Fig. 13**) the 7^{th} joint deviation from the desired trajectories had not been compensated by the 5^{th} joint motion as it was expected. The last is due to one main reason. The kernel trajectory has been generated in advance, before starting the real control and has not been updated online, by taking the measured joint values each 2 or 3 sampling intervals.

Further, the results showing how the robot state has been affected by applying some force from the left side

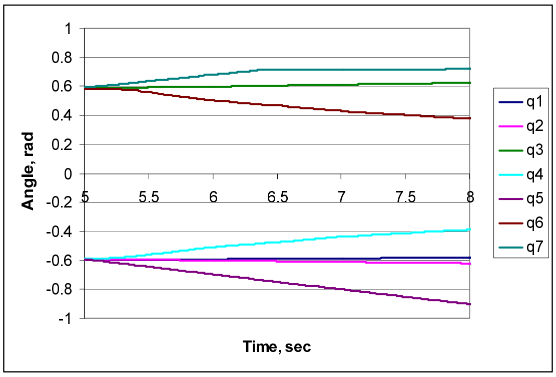

Fig. 13. Measured Kernel trajectories.

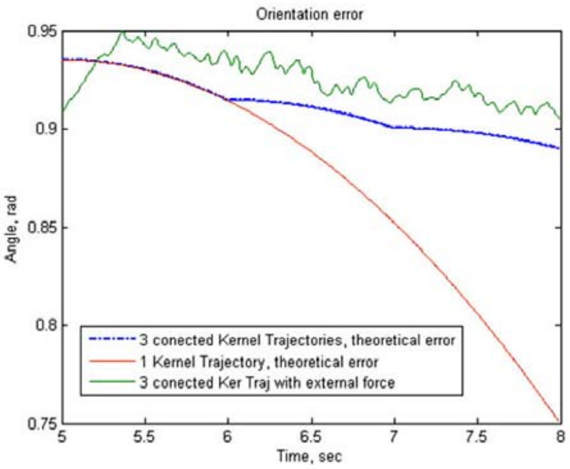

Fig. 14. Measured and theoretical orientation trajectory.

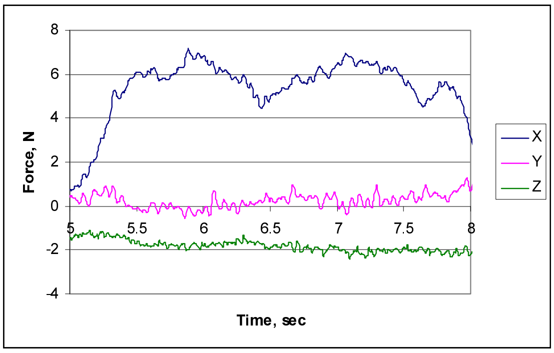

Fig. 15. Measured external force.

of the 7^{th} link are shown (**Fig. 14** and **Fig. 15**).

In this case the orientation error is very small -0.0045 rad. The end-effector state has been maintained. The orientation and position errors are close to the theoretically estimated values. Considering the results from the real experiments we can conclude that the robot end-effector could resist different external forces based on the redundancy features, by simply generate desired joint trajectories belonging to the Null space or Kernel. They are easy to be obtained in real time and do not require complex calculations. The control scheme is realized in the joint space.

6. Conclusions and Future Work

A novel approach for redundant robots force control has been proposed in the work. This strategy allows the robot end-effector to resist to external force by generating such desired joints trajectories in the configuration space, which do not affect the motion in the working zone. To achieve this purpose the features of the so called Null space or Kernel have been used. The configuration space has been divided into 4 independent subsets, each responsible for different motion. One of these 4 sets represents the orientation and position homomorphisms Kernels intersection. By using its vector elements we can generate the desired Kernel trajectories, responsible to maintain the robot end-effector state, while some "hidden" motion occurs in the joint space. Further if this trajectories are generated online, during the real control, on the base of the measured values of the joints, and some joint position (angles) has been affected by the applied external force, in that case this deviation can be compensated by the motion of some pair joint, by solving (3). In our case the Kernel trajectories were generated in advance in order to simplify the control scheme, which caused some deviations of the end-effector state for some of the cases, but although that it was proved the method is efficient to make the robot end-effector resist to some external influence, when we have constraint tasks. This control features could be used for instance if we have a redundant robot, which has to make a hole into some object with variable density (for instance drilling bones in orthopedic operations). Then the end-effector position has to be maintained stable and unchanged while the drill will penetrate the object with variable density. If the robot has more degrees of freedom, better possibilities exist to compensate forces from different directions. The last robot feature depends also on the joints displacement in the kinematics chain. This approach is easy to be applied, because the necessary mathematical tools are based on algebraic system of equations, eigenvectors and eigenvalues. To constrain the end-effector motion, no additional penalty functions are needed as in the case of the conventional methods. No pseudo-inverse Jacobian calculations are needed; the desired trajectories are generated in the joint space. To compute the desired Kernel trajectories we have chosen a linear law of variation of the joints, for simplicity. Some other cases should be investigated in the future. To compute the control torque we have chosen the servo-control method with standard trajectories [15], which is a modified computed torque method. Of course we are not restricted only to this method. The key in this control scheme is the Kernel trajectories generation. The last, together with the proof of the possibilities of force compensation by using the redundant robot features, is *the main purpose* of the work. Of course, lots of other questions arise, related to redundant robots control – the singular problem, the stability of the motion, the case when the redundancy increases and so on. No doubt all these points are very interesting not only for truly investigating the redundant robots but also for separate analyses. The answers and the results are surely expected to be interesting and remain subject of future work. It seems good to point out some advantages of the proposed control method. First, a new idea is proposed for exploring the quality features of the robots with redundancy – for the remaining it does not work. For that some not standard techniques and tools have been used – homomorphisms, groups and factor-groups, manifolds, orthogonal fibrations and kernel trajectories. They help to describe some redundancy characteristics more clearly and extract a part of quality possibilities of the redundant robots with the aim to apply them in practical applications. In the work kernel trajectories practically assure a redistribution of the inner passive forces in the redundant system, keeping the established state in the working zone. This was confirmed by the experiments. As a further investigation the role of the kernel trajectories in the sense of *applying a desired force* in the contact tasks must be clarified.

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