# Joint Trajectory Planning Based on Minimum Euclidean Distance of Joint Angles of a Seven-Degrees-of-Freedom Manipulator for a Sequential Reaching Task 

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#### Abstract

When a nuclear power disaster occurs at a nuclear power plant, it is hazardous for humans to enter the plant. If robots could remove radioactive substances adhering to a plane such as a plant wall, humans would be able to enter the plant to investigate the situation and to work. In this study, to efficiently remove radioactive substances from a wall with a manipulator, we examined joint trajectory planning based on the minimum Euclidean distance of joint angles of a seven-degrees-of-freedom (7-DOF) serial link manipulator for a sequential reaching task on a plane. We demonstrate the planning for the sequential reaching task, which is an iterative point-to-point reaching movement between positions on a plane. The joint angles for each target position were obtained based on the inverse kinematics for an arm angle, and the optimal arm angles within the constraints of the joint angles were computed by the sequential quadratic programming method. The optimal trajectories for the arm angles were compared with the trajectories of the joint angles that were the eight inverse kinematic solutions for a fixed arm angle. The result showed that through optimal planning, an efficient trajectory within the movable ranges of the joint angles could be obtained for the sequential reaching task.


Keywords: optimal planning, sequential reaching task, 7-DOF manipulator, minimum Euclidean distance, movable ranges

## 1. Introduction

When a nuclear disaster occurs at a nuclear power plant, the radiation dose inside the nuclear power plant is high. It is then hazardous for humans to enter the plant. If robots could remove radioactive substances adhering to a plane such as a plant wall, humans would be able to enter the nuclear power plant and investigate the situation or work.

Radioactivity can be decontaminated using a device that blows high-pressure water or abrasives, or irradiates
a laser onto a surface such as a wall. The device is attached to an end effector of a manipulator and can remove radioactive substances from various places when the manipulator is moved [1-3]. The laser decontamination device and the manipulator are generally mounted on a mobile robot with wheels or crawlers. A laser decontamination device that measures the distance to the surface and simultaneously removes radioactive material can decontaminate a non-uniform or non-smooth surface [3]. The plane of a wall can be obtained by the Hough transformation or by random sample consensus from point cloud data, which can be obtained by a 3D laser scanner.
Although manipulators with redundancy are excellent for singularity avoidance, collision avoidance, and maintaining high manipulability, there are infinite combinations of joint angles that realize the position and orientation of the end effector; the inverse kinematic solution of the manipulator cannot be uniquely determined [4]. Bicriteria velocity minimization based on a primal-dual neural network has been proposed to resolve the redundancy problem for robot manipulators [5]. The study demonstrated the tasks of circular path tracking and straight path tracking considering the limitations of the angles and angular velocities of a manipulator (PUMA560). On the other hand, solutions to the inverse kinematics of a seven-degrees-of-freedom (7-DOF) serial link manipulator using the arm angle have been proposed [6]. The arm angle is the angle between the arm plane and a reference plane, and it is a scalar variable by which the redundancy of the 7-DOF manipulator is parameterized to determine the joint angles for a given end effector position and orientation [4]. When the arm angle of the manipulator is given, there are eight solutions according to inverse kinematics. If we can determine the optimal trajectory of arm angles that suppress the change in joint angles of the manipulator within the movable ranges of the joint angles, it would be possible to remove the radioactive substances efficiently.
In this study, we examine joint trajectory planning based on the minimum Euclidean distance of joint angles of a 7-DOF serial link manipulator for a sequential reaching task in a plane. We demonstrate the planning for a sequential reaching task, which is an iterative

Table 1. Link parameters for SIA-20.


Fig. 1. 7-DOF serial link manipulator.
point-to-point reaching movement between positions on a plane. The joint angles for the target position are obtained based on the inverse kinematics for the arm angle, and the optimal arm angles with the constraints of the joint angles are computed by the sequential quadratic programming method. The optimal trajectories of the joint angles are then compared with the trajectories of the joint angles that are the eight inverse kinematic solutions for a fixed arm angle. This paper is organized as follows. Section 2 describes the kinematics of the 7-DOF manipulator, the task, the inverse kinematic solutions, and the optimal planning based on the minimum Euclidean distance. Section 3 presents the results of both the optimal planning and the inverse kinematics. Finally, Section 4 concludes the paper.

## 2. Method

We computed optimal trajectories based on the minimum Euclidean distance of the joint angles of a 7-DOF serial link manipulator within the movable ranges of the manipulator joints for a sequential reaching task. The optimal trajectories of the joint angles were compared with the trajectories of the joint angles that were the eight inverse kinematic solutions for a fixed arm angle.

### 2.1. Kinematics of the $\mathbf{7 - D O F}$ Manipulator

We used a Spherical-Revolute-Spherical type manipulator as the 7-DOF serial link manipulator (Fig. 1). The shoulder joint is a spherical joint consisting of $\theta_{1}, \theta_{2}$, and $\theta_{3}$, the elbow joint is a revolute joint consisting of $\theta_{4}$, and the wrist joint is a spherical joint consisting of $\theta_{5}, \theta_{6}$, and $\theta_{7}$. The coordinate system of joints $\Sigma_{i}(i=1,2, \ldots, 7)$ is fixed to each joint $\theta_{i}$. The base coordinate system $\Sigma_{0}$ is fixed to the root of the manipulator, and the coordinate system $\Sigma_{7}$ is set at the endpoint of the manipulator. Tables 1 and 2 show the link parameters based on the Denavit-Hartenberg rules [7] and the movable ranges of the joint angles for the 7-DOF serial link manipulator (SIA-20, Yaskawa Electric Corporation).

The position ${ }^{0} \boldsymbol{x}_{7}$ and orientation ${ }^{0} \boldsymbol{R}_{7}$ of the end effector

| $i$ | $a_{i}[\mathrm{~m}]$ | $\alpha_{i}[\mathrm{rad}]$ | $d_{i}[\mathrm{~m}]$ | $\theta_{i}[\mathrm{rad}]$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | $-\pi / 2$ | 0.36 | $\theta_{1}$ |
| 2 | 0 | $\pi / 2$ | 0 | $\theta_{2}$ |
| 3 | 0 | $-\pi / 2$ | 0.42 | $\theta_{3}$ |
| 4 | 0 | $\pi / 2$ | 0 | $\theta_{4}$ |
| 5 | 0 | $-\pi / 2$ | 0.40 | $\theta_{5}$ |
| 6 | 0 | $\pi / 2$ | 0 | $\theta_{6}$ |
| 7 | 0 | 0 | 0.111 | $\theta_{7}$ |

Table 2. Movable ranges of joint angles for SIA-20.

| $i$ | $\theta_{i}^{\text {min }}\left[{ }^{\circ}\right]$ | $\theta_{i}^{\max }\left[{ }^{\circ}\right]$ |
| :---: | :---: | :---: |
| 1 | -180 | 180 |
| 2 | -110 | 110 |
| 3 | -170 | 170 |
| 4 | -130 | 130 |
| 5 | -180 | 180 |
| 6 | -110 | 110 |
| 7 | -180 | 180 |

are expressed as follows [6]:

$$
\begin{align*}
& { }^{0} \boldsymbol{x}_{7}={ }^{0} \boldsymbol{l}_{b s}+{ }^{0} \boldsymbol{R}_{\Theta}{ }^{0} \boldsymbol{R}_{3}^{o}\left\{{ }^{3} \boldsymbol{l}_{s e}+{ }^{3} \boldsymbol{R}_{4}\left({ }^{4} \boldsymbol{l}_{e w}+{ }^{4} \boldsymbol{R}_{7}{ }^{7} \boldsymbol{l}_{w t}\right)\right\}  \tag{1}\\
& { }^{0} \boldsymbol{R}_{7}={ }^{0} \boldsymbol{R}_{\Theta}{ }^{0} \boldsymbol{R}_{3}^{o 3} \boldsymbol{R}_{4}^{4} \boldsymbol{R}_{7} . . . . . . . . . . . \tag{2}
\end{align*}
$$

where ${ }^{i} \boldsymbol{R}_{j}$ is the rotation from the coordination $\Sigma_{j}$ to the coordination $\Sigma_{i},{ }^{0} \boldsymbol{R}_{\Theta}$ is the rotation of the arm angle $\Theta$, which is the rotation around the axis connecting the shoulder joint and the wrist joint, and ${ }^{i} \boldsymbol{R}_{j}^{o}$ is the rotation when the arm plane corresponds to the reference plane (i.e., $\Theta=0$ ), ${ }^{0} \boldsymbol{l}_{b s}=\left[0,0, d_{1}\right]^{T},{ }^{3} \boldsymbol{l}_{s e}=\left[0,-d_{3}, 0\right]^{T}$, ${ }^{4} \boldsymbol{l}_{e w}=\left[0,0, d_{5}\right]^{T}$, and ${ }^{7} \boldsymbol{l}_{w t}=\left[0,0, d_{7}\right]^{T}$. The rotation between coordinate systems $\Sigma_{i-1}$ and $\Sigma_{i}$ is given by

$$
{ }^{i-1} R_{i}=\left[\begin{array}{ccc}
\cos \theta_{i} & -\sin \theta_{i} \cos \alpha_{i} & \sin \theta_{i} \sin \alpha_{i}  \tag{3}\\
\sin \theta_{i} & \cos \theta_{i} \cos \alpha_{i} & -\cos \theta_{i} \sin \alpha_{i} \\
0 & \sin \alpha_{i} & \cos \alpha_{i}
\end{array}\right]
$$

The rotation ${ }^{0} \boldsymbol{R}_{\Theta}$ of the arm angle is given by

$$
\begin{equation*}
{ }^{0} \boldsymbol{R}_{\Theta}=\boldsymbol{I}_{3}+\sin \Theta\left[{ }^{0} \boldsymbol{u}_{s w} \times\right]+(1-\cos \Theta)\left[{ }^{0} \boldsymbol{u}_{s w} \times\right]^{2} \tag{4}
\end{equation*}
$$

where $\boldsymbol{I}_{3}$ is the identity matrix, ${ }^{0} \boldsymbol{u}_{s w}$ is the unit vector of ${ }^{0} \boldsymbol{x}_{s w}$, and $\left[{ }^{0} \boldsymbol{u}_{s w} \times\right]$ is the skew-symmetric matrix corresponding the cross product with the vector ${ }^{0} \boldsymbol{u}_{s w}$. The vector ${ }^{0} \boldsymbol{x}_{s w}$ from the shoulder joint to the wrist joint is expressed as follows:

### 2.2. Task

Figure 2 shows a sequential reaching task for target points on a target plane. In the demonstration, the target plane was assumed to be on the $x z$-plane at $y=0.6 \mathrm{~m}$. The ranges of the $x$ and $z$ positions on the plane were $-0.4 \leq x \leq 0.4$ and $0.2 \leq z \leq 1.0$, respectively. The target points $\boldsymbol{x}_{d}^{k}=\left(x_{d}^{k}, y_{d}^{k}, z_{d}^{k}\right)(k=1,2, \ldots, N)$ are evenly spaced because the laser decontaminator that is attached to the end effector emits a laser that covers approximately 100


Fig. 2. Sequential reaching task on a plane. The circle points show the target points. The gray solid and dashed lines show routes based on the horizontal direction and the vertical direction, respectively. The squares show the Spherical-Revolute-Spherical joints of the 7-DOF serial link manipulator.


Fig. 3. Areas irradiated by the laser in a target plane. The filled area shows the area that the laser can irradiate at one time.
$\times 100 \mathrm{~mm}^{2}$ (Fig. 3). Then, the target points of the end effector were $(x, y, z)=(-0.35+0.1 \times m, 0.6,0.25+0.1 \times$ $n)(m, n=0,1, \ldots, 7)$, and the number of target points $N$ was 64 points in 8 rows and 8 columns. The starting point for the task was set at $(0.35,0.6,0.95)$. The routes of the sequential reaching motion were based on vertical and horizontal directions (Fig. 2). To irradiate the whole of each target area on the plane, the $z$-axis of the end effector has to be perpendicular to the plane, and the posture of the end effector has to remain unchanged. We assumed that the $y$-axis of the end effector was oriented in the direction of the $z$-axis of $\sum_{0}$. Then, the orientation of the end effector was as follows:

$$
{ }^{0} \boldsymbol{R}_{7}^{d}=\left[\begin{array}{ccc}
-1 & 0 & 0  \tag{6}\\
0 & 0 & 1 \\
0 & 1 & 0
\end{array}\right] .
$$

### 2.3. Inverse Kinematics of the 7-DOF Manipulator

We followed the method of inverse kinematics for the 7-DOF manipulator proposed in [6]. If an arm angle is given, there are eight inverse kinematic solutions for the 7-DOF manipulator. Thus, the solutions by inverse kinematics are expressed as follows:

$$
\begin{align*}
& \theta_{1}=\arctan 2\left(-a_{s 22} \sin \Theta-b_{s 22} \cos \Theta-c_{s 22},\right. \\
& \left.-a_{s 12} \sin \Theta-b_{s 12} \cos \Theta-c_{s 12}\right)  \tag{7}\\
& \theta_{2}=\arctan 2\left(\sqrt{\sum_{i=1}^{2}\left(a_{s i 2} \sin \Theta+b_{s i 2} \cos \Theta+c_{s i 2}\right)^{2}},\right. \\
& \left.-a_{s 32} \sin \Theta-b_{s 32} \cos \Theta-c_{s 32}\right)  \tag{8}\\
& \theta_{3}=\arctan 2\left(a_{s 33} \sin \Theta+b_{s 33} \cos \Theta+c_{s 33}\right. \text {, } \\
& \left.-a_{s 31} \sin \Theta-b_{s 31} \cos \Theta-c_{s 31}\right)  \tag{9}\\
& \theta_{4}=\gamma_{1} \arccos \left(\frac{\| \|^{0} \boldsymbol{x}_{s w} \|^{2}-d_{3}^{2}-d_{5}^{2}}{2 d_{3} d_{5}}\right)  \tag{10}\\
& \theta_{5}=\arctan 2\left(\gamma_{3}\left(a_{w 23} \sin \Theta+b_{w 23} \cos \Theta+c_{w 23}\right)\right. \text {, } \\
& \left.\gamma_{3}\left(a_{w 13} \sin \Theta+b_{w 13} \cos \Theta+c_{w 13}\right)\right)  \tag{11}\\
& \theta_{6}=\gamma_{3} \arctan 2\left(\sqrt{\sum_{i=1}^{2}\left(a_{w i 3} \sin \Theta+b_{w i 3} \cos \Theta+c_{w i 3}\right)^{2}}\right. \text {, } \\
& \left.a_{w 33} \sin \Theta+b_{w 33} \cos \Theta+c_{w 33}\right) \\
& \theta_{7}=\arctan 2\left(\gamma_{3}\left(a_{w 32} \sin \Theta+b_{w 32} \cos \Theta+c_{w 32}\right)\right. \text {, } \\
& \left.\gamma_{3}\left(-a_{w 31} \sin \Theta-b_{w 31} \cos \Theta-c_{w 31}\right)\right) \tag{13}
\end{align*}
$$

where $\gamma_{1}$ and $\gamma_{3}$ are parameters that indicate the different inverse kinematic solutions, and $a_{s i j}, b_{s i j}, c_{s i j}, a_{w i j}, b_{w i j}$, and $c_{w i j}$ are the $(i, j)$ elements of the matrices $\boldsymbol{A}_{s}, \boldsymbol{B}_{s}, \boldsymbol{C}_{s}$,

Table 3. Combinations of parameters for the different inverse kinematic solutions.

| Solution No. | $\gamma_{1}$ | $\gamma_{2}$ | $\gamma_{3}$ |
| :---: | ---: | ---: | ---: |
| 1 | +1 | +1 | +1 |
| 2 | +1 | +1 | -1 |
| 3 | +1 | -1 | +1 |
| 4 | +1 | -1 | -1 |
| 5 | -1 | +1 | +1 |
| 6 | -1 | +1 | -1 |
| 7 | -1 | -1 | +1 |
| 8 | -1 | -1 | -1 |

$\boldsymbol{A}_{w}, \boldsymbol{B}_{w}$, and $\boldsymbol{C}_{w}$, respectively;

$$
\begin{equation*}
\boldsymbol{A}_{s}=\left[{ }^{0} \boldsymbol{u}_{s w} \times\right]^{0} \boldsymbol{R}_{3}^{o} \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
\boldsymbol{B}_{s}=-\left[{ }^{0} \boldsymbol{u}_{s w} \times\right]^{2}{ }^{0} \boldsymbol{R}_{3}^{o} \tag{15}
\end{equation*}
$$

$$
\begin{equation*}
\boldsymbol{C}_{s}=\left[{ }^{0} \boldsymbol{u}_{s w}{ }^{0} \boldsymbol{u}_{s w}^{T}\right]{ }^{0} \boldsymbol{R}_{3}^{o} \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\boldsymbol{A}_{w}={ }^{3} \boldsymbol{R}_{4}^{T} \boldsymbol{A}_{s}^{T}{ }^{0} \boldsymbol{R}_{7}^{d} \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
\boldsymbol{B}_{w}={ }^{3} \boldsymbol{R}_{4}^{T} \boldsymbol{B}_{s}^{T}{ }^{0} \boldsymbol{R}_{7}^{d} \tag{18}
\end{equation*}
$$

$$
\begin{equation*}
\boldsymbol{C}_{w}={ }^{3} \boldsymbol{R}_{4}^{T} \boldsymbol{C}_{s}^{T}{ }^{0} \boldsymbol{R}_{7}^{d} \tag{19}
\end{equation*}
$$

The joint angles $\theta_{1}^{\circ}, \theta_{2}^{\circ}, \theta_{3}^{\circ}$ when the arm plane corresponds to the reference plane are as follows:

$$
\begin{align*}
\theta_{1}^{\circ}= & \arctan 2\left(x_{s w 1},-x_{s w 2}\right) \\
& -\arctan 2\left(\gamma_{2} \sqrt{x_{s w 1}^{2}+x_{s w 2}^{2}}, 0\right) \ldots  \tag{20}\\
\theta_{2}^{\circ}= & \arctan 2\left(b_{1}, b_{2}\right) \\
& -\arctan 2\left(\gamma_{2} \sqrt{b_{1}^{2}+b_{2}^{2}-x_{s w 3}^{2}},-x_{s w 3}\right)  \tag{21}\\
\theta_{3}^{\circ}= & 0 . . . . . . . . . . . . . . \tag{22}
\end{align*}
$$

where ${ }^{0} \boldsymbol{x}_{s w}=\left[x_{s w 1}, x_{s w 2}, x_{s w 3}\right]^{T}, \quad b_{1}=d_{5} \sin \theta_{4}, \quad b_{2}=$ $-d_{3}-d_{5} \cos \theta_{4}$, and $\gamma_{2}$ is a parameter that indicates the different inverse kinematic solutions. The different inverse kinematic solutions are represented by the combinations of $\gamma_{1}= \pm 1, \gamma_{2}= \pm 1$, and $\gamma_{3}= \pm 1$ (Table 3).

### 2.4. Constrained Optimization Problem

We computed the optimal trajectories of arm angles $\boldsymbol{\Theta}$ for each inverse kinematic solution (eight solutions), and then obtained the optimal trajectories of the joint angles from the trajectories of the arm angles. The evaluation function based on the minimum Euclidean distance of the joint angles takes the following form:

$$
\begin{equation*}
E(\boldsymbol{\Theta})=\sum_{k=1}^{N-1}\left\|\boldsymbol{\theta}^{k+1}-\boldsymbol{\theta}^{k}\right\| \tag{23}
\end{equation*}
$$

where $\boldsymbol{\Theta}=\left[\Theta^{1}, \Theta^{2}, \ldots, \Theta^{k}, \ldots, \Theta^{N}\right]^{T}, k$ indicates the index of the target point, $\|\boldsymbol{x}\|$ denotes the Euclidean norm of $\boldsymbol{x}$, and $\boldsymbol{\theta}^{k}=\left[\theta_{1}^{k}, \theta_{2}^{k}, \ldots, \theta_{7}^{k}\right]^{T}$. The inequality constraints of the optimization problem are as follows:

$$
\begin{align*}
-\pi & \leq \Theta^{k} \leq \pi  \tag{24}\\
\theta_{i}^{\min } & \leq \theta_{i}^{k} \leq \theta_{i}^{\max } \tag{25}
\end{align*}
$$

where $\theta^{\min }$ and $\theta^{\max }$ show the lower bound and upper bound of each joint angle, respectively. Eqs. (24) and (25) show the constraints for the arm angle and the movable ranges of the joint angles, respectively. Note that because the joint angle $\theta_{4}$ is not dependent on the arm angle (Eq. (10)), the constraint of the joint angle $\theta_{4}^{k}$ cannot be included in the constrained optimization problem (Eq. (25)). When $\theta_{4}^{k}$ does not satisfy the movable range of the joint angle, there is no inverse kinematic solution.

The initial values of the arm angles $\boldsymbol{\Theta}$ were all set to $\pi / 4 \mathrm{rad}$. The optimal arm angles were computed by the sequential quadratic programming method. The method was executed using SNOPT (Stanford Business Software, Inc.) [8].

## 3. Result

Tables 4 and 5 show the Euclidean distances and constraints of the inverse kinematic solutions for the horizontal route and the vertical route, respectively. The inverse kinematic solutions were obtained when the arm angles at the target points were set to $\pi / 4$. Half of the solutions did not satisfy the constraints. Although in the horizontal route the eighth solution satisfied the constraints and covered the smallest Euclidean distance ( $E=39.369$ rad), the third solution in the vertical route covered a smaller Euclidean distance ( $E=34.660 \mathrm{rad}$ ).

Tables 6 and 7 show the Euclidean distances and constraints of the optimal solutions based on the minimum Euclidean distance of the joint angles for the horizontal route and the vertical route, respectively. As the result of the constrained optimization, all optimal solutions satisfied the constraints of the joint angles. Furthermore, the sixth optimal solution along the horizontal route covered the smallest Euclidean distance ( $E=18.246 \mathrm{rad}$ ) and was smaller than the inverse kinematic solutions. Note that although the constraints were satisfied, there were also cases where the Euclidean distance was large. By selecting the optimal solution with the smallest Euclidean distance among the eight solutions, it was possible to obtain the optimal angles for efficiently removing radioactive substances.

Figure 4 shows the trajectories of the joint angles of solution No. 6 along the horizontal route. In the inverse kinematic solution, $\theta_{2}$ did not satisfy the constraint, but the optimal solution satisfied the constraint. Furthermore, the change at each joint angle of the optimal solution was smaller than that of inverse kinematics; that is, the Euclidean distance of the optimal solution was smaller than that of inverse kinematics. Fig. 5 shows the trajectories of the joint angles of solution No. 7 along the horizontal route. Although the constraints were satisfied, the change in each joint angle of the optimal solution was larger than that of inverse kinematics; that is, the Euclidean distance of the optimal solution was larger than that of inverse kinematics. Because the optimized result was not always right, it would be necessary to select the best solution out of the eight optimal solutions.

Table 4. Euclidean distances and constraints of the inverse kinematic solutions for the horizontal route.

| Solution No. | Value [rad] | Constraints |
| :---: | ---: | :---: |
| 1 | 73.489 | $\circ$ |
| 2 | 64.610 | $\circ$ |
| 3 | 43.017 | $\circ$ |
| 4 | 67.554 | $\times$ |
| 5 | 21.358 | $\times$ |
| 6 | 21.358 | $\times$ |
| 7 | 67.554 | $\times$ |
| 8 | 39.369 | $\circ$ |

Table 6. Euclidean distances and constraints of the optimal solutions for the horizontal route.

| Solution No. | Value [rad] | Constraints |
| :---: | ---: | :---: |
| 1 | 64.097 | $\circ$ |
| 2 | 70.586 | $\circ$ |
| 3 | 22.524 | $\circ$ |
| 4 | 72.602 | $\circ$ |
| 5 | 47.618 | $\circ$ |
| 6 | 18.246 | $\circ$ |
| 7 | 187.105 | $\circ$ |
| 8 | 28.681 | $\circ$ |

Table 5. Euclidean distances and constraints of the inverse kinematic solutions for the vertical route.

| Solution No. | Value [rad] | Constraints |
| :---: | ---: | :---: |
| 1 | 62.225 | $\circ$ |
| 2 | 88.833 | $\circ$ |
| 3 | 34.660 | $\circ$ |
| 4 | 48.941 | $\times$ |
| 5 | 19.926 | $\times$ |
| 6 | 19.926 | $\times$ |
| 7 | 48.941 | $\times$ |
| 8 | 64.988 | $\circ$ |

Table 7. Euclidean distances and constraints of the optimal solutions for the vertical route.

| Solution No. | Value [rad] | Constraints |
| :---: | ---: | :---: |
| 1 | 67.451 | $\circ$ |
| 2 | 79.829 | $\circ$ |
| 3 | 20.749 | $\circ$ |
| 4 | 74.308 | $\circ$ |
| 5 | 58.750 | $\circ$ |
| 6 | 20.749 | $\circ$ |
| 7 | 64.192 | $\circ$ |
| 8 | 51.485 | $\circ$ |


(a) Joint angle $\theta_{1}$

(d) Joint angle $\theta_{4}$

(b) Joint angle $\theta_{2}$

(e) Joint angle $\theta_{5}$

(c) Joint angle $\theta_{3}$

(f) Joint angle $\theta_{6}$

(g) Joint angle $\theta_{7}$

Fig. 4. Trajectories of joint angles of solution No. 6 along the horizontal route. The black and gray dotted lines show the optimal solution based on the minimum Euclidean distance and the inverse kinematic solution, respectively. The dashed lines show the lower bounds and the upper bounds of the joint angles. Note that the fourth joint angle is the same in the inverse kinematic solution and the optimal solution because the joint angle is independent of the arm angle.


Fig. 5. Trajectories of joint angles of solution No. 7 along the horizontal route. The black and gray dotted lines show the optimal solution based on the minimum Euclidean distance and the inverse kinematic solution, respectively. The dashed lines show the lower bounds and the upper bounds of the joint angles. Note that the fourth joint angle is the same in the inverse kinematic solution and the optimal solution because the joint angle is independent of the arm angle.

## 4. Conclusion

We proposed optimal trajectory planning based on the minimum Euclidean distance of joint angles of a 7-DOF serial link manipulator within the movable ranges of the manipulator joints for a sequential reaching task. As the result of the optimal planning, the optimal solution covered the smallest Euclidean distance and satisfied the movable ranges of the joint angles. From the result, the optimal planning obtained an efficient trajectory for the joint angles of the 7-DOF serial link manipulator for the sequential reaching task. Furthermore, because the optimizations for the different inverse kinematic solutions are irrelevant, the optimal solution could be efficiently obtained by parallel processing such as with OpenMP. In future work, in the case where a wall is quite uneven and not smooth, or when multiple planes are included, it will also be necessary to consider the constraint that the manipulator does not contact the wall.

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