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Paper:

Design of Modified Repetitive Controller for T–S Fuzzy Systems

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A repetitive controller contains a pure-delay positive-feedback loop that makes it difficult to stabilize a strictly proper system. A low-pass filter is inserted in a repetitive controller to relax the stability condition of the modified repetitive-control system at the cost of degrading the tracking performance. In this study, a modified repetitive-control approach is developed, which reaches a balance between the stability and tracking performance for a class of affine nonlinear systems based on the Takagi–Sugeno fuzzy model. First, a 2D model is established to adjust continuous control and discrete learning actions preferentially induced by exploiting the 2D property in a repetitive-control process. Then, the Lyapunov stability theory and 2D system theory are used to derive a sufficient stability condition in the form of linear matrix inequalities to design parallel-distributed-compensation-based state-feedback controllers. Finally, an application-oriented example is used, and a comparison is performed to show that an extra variable is introduced such that the developed method has a better tracking performance.

Keywords: affine nonlinear systems, modified repetitive control, Takagi–Sugeno fuzzy model, two-dimensional model, parallel distributed compensation

1. Introduction

There exist many tasks of tracking periodic references or rejecting periodic disturbances in industrial applications [1, 2]. Repetitive control (RC), which has been proposed by the team of professor Michio Nakano, meets the goal of tracking periodic signals with a high accuracy in a short time. It tracks or rejects periodic references effectively without a steady-state error for linear systems as it is based on the internal model theory [3, 4]. However, it is difficult to stabilize RC systems that introduce a pure-delay positive-feedback line. Modified repetitive control (MRC) was proposed to improve the stability conditions by incorporating a low-pass filter in the delay positive-feedback loop [5]. Such a low-pass filter guarantees the system stability at the cost of the tracking performance of high-frequency signals. Moreover, the relative degree of the system is not zero, which is the case in the majority of control engineering systems.

There are two types of actions in an RC process: continuous control within a period and discrete learning between periods. Wu et al. [6, 7] presented a 2D model for depicting an RC system. Thus, the system control issue of an RC system is converted into a stability issue of a 2D system. Moreover, the continuous–discrete 2D hybrid model takes into consideration the effect of the two actions and is more in line with the self-learning nature of an RC process. The continuous–discrete 2D model and 2D control law facilitate the regulation of the learning and control preferentially. Thus, the system exhibits good dynamic responses when it is stable.

It should be noted that these researches have been mainly conducted in the linear system framework. However, the majority of physical systems in industrial processes are nonlinear, and nonlinearities seriously affect the system performance. It is difficult to solve the issue of tracking periodic signals in nonlinear systems using the linear RC theory. In recent years, the study of RC in nonlinear systems has drawn increasing attention [8]. The issue of control in complex nonlinear systems is difficult to solve using conventional control theory [9, 10]. Theoretically, previously reported methods have been based on some assumptions for nonlinear systems, which are not always accurate for real systems; Thus, they have limitations. This means that the tracking problem of periodic signals in nonlinear systems remains a challenging issue.

The Takagi–Sugeno (T–S) fuzzy model relates nonlinear control to linear control using fuzzy rules. It has been used as a universal approach for representing a nonlinear system based on its universal approximation properties [11]. It utilizes linear-invariant models combined with nonlinear fuzzy membership functions that are "close" to a nonlinear system in some local regions. Linear system theory can be directly applied in order to analyze the performance of nonlinear systems. The Lyapunov stability theory is a common method for dealing with the stability analysis of fuzzy systems [12]. Sharma et al. [13] used the Lyapunov stability theory to solve the stability problem of fuzzy systems. Although the general Lyapunov method has certain limitations, it is simple and straightforward. It has been an effective method for solving the stabilization problem of T–S fuzzy systems. Thus, many scholars have conducted researches on solving fuzzy system stabilization conditions based on the Lyapunov stability theory [14, 15].

In this study, a new MRC approach is developed to track periodic signals effectively for a class of affine nonlinear systems. The general affine nonlinear system is depicted by the T–S fuzzy model with several fuzzy rules. Thus, the RC theory is directly applied to linear subsystems. Using the 2D system theory and Lyapunov stability theory, a sufficient stability condition in the form of a set of linear matrix inequalities (LMIs) is obtained to design the stabilization controllers. Lastly, an example is used, and a comparison is performed to verify the effectiveness of the method. The following are the main contributions of this paper:

- The T–S fuzzy model is used to depict a class of affine nonlinear systems. Thus, linear RC theory is directly employed to analyze the tracking performance of nonlinear systems.
- A new MRC approach that possesses both the advantages of RC and MRC is developed. The extra variable w reaches a balance between the control performance and stability. This is a great improvement as compared to that in [16].
- A 2D continuous–discrete hybrid model that adjusts two actions in an RC process preferentially is used to improve the transient performance of the system.

The rest of this paper is organized as follow: Section 2 presents the problem formulation and system description. The stability analysis and controller design are presented in Section 3. An application-oriented example is presented in Section 4. Finally, Section 5 concludes the paper.

Notation: throughout the paper, the n-dimensional Euclidean space is represented by $\mathbb{R}^n$. $\mathbb{Z}^+$ is the set of nonnegative integers. Ψ^{-1} and Ψ^{T} denote the matrix inverse and matrix transposition of Ψ, respectively. All $n \times p$ real matrices are defined by $\mathbb{R}^{n\times p}$. The notations diag$\{\dots\}$ stands for a diagonal matrix. P, symmetric and positive definite or negative definite, is denoted by $P > 0$ or $P < 0$, respectively. $*$ represents the symmetric term in a matrix. If the matrices are not stated explicitly, they are assumed to have appropriate dimensions.

2. Problem Formulation

In this paper, a general affine nonlinear system with a single input and single output is under consideration. Its dynamics are described as

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) \\ y(t) = h(x(t)) \\ x(t) = \xi(t), t \in [-\tau, 0] \end{cases} \quad \dots\dots\dots \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state of the plant, $y(t) \in \mathbb{R}$ is the system output and $u(t) \in \mathbb{R}$ is the control input. ξ is the continuous initial state function of the plant. f, g, and h represent continuous nonlinear functions.

Figure 1 [17] shows the configuration of a new MRC system based on the T–S fuzzy model. The new MRC structure contains two delay positive-feedback lines $e^{-s\mathrm{T}}$ with a low-pass filter $q(s)$ and a constant value w, which improves the tracking performance of periodic signals. Without the loss of generality, the low-pass filter is presented as

$$q(s) = \frac{\omega_c}{s + \omega_c} \quad \dots\dots\dots\dots\dots \quad (2)$$

where ω_c is the cut-off frequency. In order to track the periodic signals of all frequencies as much as possible, it is better to select ω_c as a value 5–10 times greater than the highest frequency of the given periodic references [18]. For the convenience of design and implementation, the low-pass filter is expressed as

$$q(s) = \frac{\rho\,\omega_c}{s + \omega_c}, \rho \approx 1. \quad \dots\dots\dots\dots \quad (3)$$

Thus,

$$X_q(s) = \frac{1}{1 - q(s)e^{-sT}}E(s). \quad \dots\dots\dots\dots \quad (4)$$

We then take the inverse Laplace transform of formulation (4). The state–space description of the new repetitive controller is

$$\begin{cases} \dot{x}_q(t) = -\omega_c x_q(t) + \omega_c x_q(t-T) + \omega_c e(t) + \dot{e}(t) \\ v_w(t) = w v_w(t-T) + e(t) \\ e(t) = r(t) - y(t) \\ x_q(t) = 0, \ \ t \in [-T, 0] \end{cases} \quad (5)$$

where $x_q(t)$ is the state variable. $r(t)$ and $e(t)$ are a periodic input with a period of T and the tracking error, respectively. $v_w(t)$ is the output of the repetitive controller.

Let us consider a nonlinear system (1) based on the T–S fuzzy model with r fuzzy rules.

R^i: If $\lambda_1(t)$ is F_{1i}, $\lambda_2(t)$ is F_{2i} and … and $\lambda_p(t)$ is F_{pi}, then

$$\begin{cases} \dot{x}_p(t) = A_i x_p(t) + B_i u(t) \\ y_p(t) = C_i x_p(t) \end{cases} \quad \dots\dots\dots \quad (6)$$

where R^i represents the i-th fuzzy rule for the fuzzy system, $i = 1, 2, \dots, r$. $\lambda_1(t), \lambda_2(t), \dots, \lambda_p(t)$ are known premise variables, and $F_{1i}, F_{2i}, \dots, F_{pi}$ are fuzzy sets. A_i, B_i, and C_i are known real-system matrices with compati-

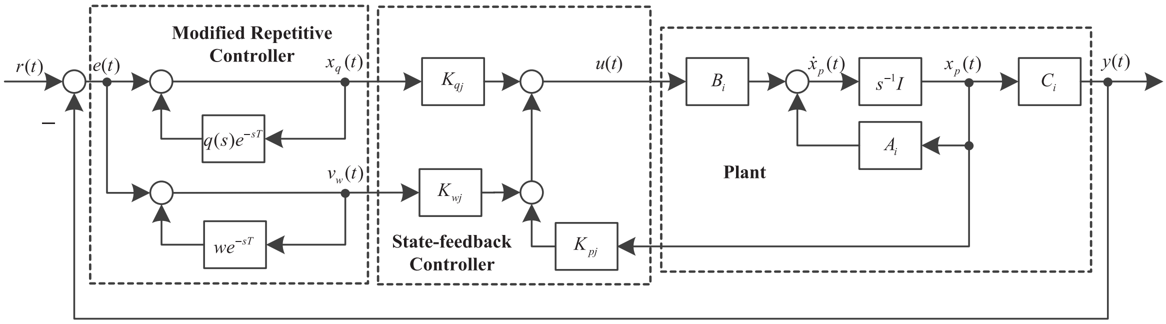


Fig. 1. Configuration of the new MRC system based on the T–S fuzzy model.

ble dimensions.

We make the following assumptions for the T–S-fuzzy-model-based system to ensure the internal stability [19]:

Assumption 1: The linear subsystem plant (A_i, B_i, C_i) is observable and controllable.

Assumption 2: The linear subsystem plant (A_i, B_i, C_i) has no zeros along the imaginary axis.

Subsequent to the singleton fuzzifier, product inference, and weighted average defuzzifier, the global model of the T–S fuzzy system (6) is depicted as follows:

$$\begin{cases} \dot{x}_p(t) = \sum_{i=1}^{r} \theta_i(\lambda(t))\,[A_i x_p(t) + B_i u(t)] \\ y_p(t) = \sum_{i=1}^{r} \theta_i(\lambda(t)) C_i x_p(t) \end{cases} \quad . \ . \ . \quad (7)$$

where $\theta_i(\lambda(t)) = (\sigma_i(\lambda(t))/\sum_{i=1}^{r}\sigma_i(\lambda(t)))$ with $\sigma_i(\lambda(t)) = \prod_{m=1}^{p} F_{mi}(\lambda(t))$, $F_{mi}(\lambda(t))$ is the grade of the membership value of $\lambda_m(t)$ in $F_{mi}, \lambda(t) = [\lambda_1(t), \lambda_2(t), \ldots, \lambda_p(t)]$. It should be noted that $\sum_{i=1}^{r}\theta_i(\lambda(t)) = 1$ and $0 < \theta_i(\lambda(t)) < 1$ for all t and $i = 1, 2, \ldots, r$.

Correspondingly, according to the parallel distributed compensation (PDC) control strategy, the state-feedback controllers are designed as

R^j: If $\lambda_1(t)$ is F_{1j}, $\lambda_2(t)$ is F_{2j} and ,..., and $\lambda_p(t)$ is F_{pj}, then

$$u(t) = K_{pj} x_p(t) + K_{qj} x_q(t) + K_{wj} v_w(t) \quad . \ . \ . \quad (8)$$

where K_{pj}, K_{qj}, and K_{wj} are controller gains. The overall output of the fuzzy controller is then represented as

$$u(t) = \sum_{j=1}^{r} \theta_j(\lambda(t))[K_{pj} x_p(t) + K_{qj} x_q(t) + K_{wj} v_w(t)]. \quad (9)$$

The design problem of the nonlinear system is to find the controller gains with the control law (9) to stabilize the system shown in **Fig. 1** and ensure a satisfactory tracking performance. The reference input $r(t)$ is set as zero. Furthermore, in order to simplify the complexity of design and computation to a great extent, all the system output matrices are assumed to be the same, i.e., $C_1 = C_2 = \cdots = C_i = C$ [20]. Hence, the derivative of the error is simplified as $\dot{e}(t) = -\dot{y}_p(t) = -C\dot{x}_p(t)$.

Moreover, by taking advantage of the 2D property in an RC process, that is, the control action during a period and learning action between periods, a vector-valued continuous-time signal $\zeta(t)$ is converted into a function-valued discrete-time sequence $\zeta_k(\tau)$ using a lifting technique [21]. τ and k are the time-continuous variable within a period and the number of discrete variables between periods, respectively. The time axis t is sliced into intervals of period T where $t = kT + \tau$ with $k \in \mathbb{Z}^+, \tau \in [0, T]$. Thus, we obtain the equation $\zeta(t) = \zeta_k(\tau) = \zeta(k, \tau)$. Then, Eqs. (5) and (7) yield the 2D model of the MRC for the T–S fuzzy system in **Fig. 1**.

$$\begin{aligned} \begin{bmatrix} \dot{\varphi}(k,\tau) \\ v_w(k,\tau) \end{bmatrix} &= \sum_{i=1}^{r} \mu_i(z(k,\tau)) \begin{bmatrix} \bar{A} & 0 \\ \bar{C} & w \end{bmatrix} \begin{bmatrix} \varphi(k,\tau) \\ v_w(k-1,\tau) \end{bmatrix} \\ &+ \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi(k,\tau - T) \\ v_w(k-2,\tau) \end{bmatrix} + \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix} u(k,\tau) \end{aligned} \quad . \quad (10)$$

where

$$\varphi(k,\tau) = [\ x_p^{\mathrm{T}}(k,\tau) \quad x_q^{\mathrm{T}}(k,\tau)\ ]^{\mathrm{T}},$$

$$\bar{A} = \begin{bmatrix} A_i & 0 \\ -\omega_c C - CA_i & -\omega_c I \end{bmatrix}, \ A_d = \begin{bmatrix} 0 & 0 \\ 0 & \omega_c I \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B_i \\ -CB_i \end{bmatrix}, \ \bar{C} = [\ -C \quad 0\ ].$$

The control law of the time domain space (Eq. (9)) represents the overall effects of learning and control. It is rewritten in the form of a 2D space (Eq. (11)) to adjust the learning and control. Thus, the tuning parameters $\tilde{K}_{pj}$, K_{qj}, and K_{wj} allow us to regulate the two actions preferentially, thereby enhancing the tracking performance. This is an obvious advantage over the 1D methods. However, we cannot adjust them completely independently as the low-pass filter and the additional loop mix learning and control.

$$\begin{aligned} u(k,\tau) &= \sum_{j=1}^{r} \theta_j(\lambda(k,\tau))[K_{pj} x_p(k,\tau) + K_{qj} x_q(k,\tau) \\ &\quad + K_{wj} v_w(k,\tau)] \\ &= \sum_{j=1}^{r} \theta_j(\lambda(k,\tau)) \left\{ \begin{bmatrix} \tilde{K}_{pj} & K_{qj} \end{bmatrix} \varphi(k,\tau) \right. \\ &\quad \left. + K_{wj} w v_w(k-1,\tau) \right\} \end{aligned} \quad (11)$$

where $\tilde{K}_{pj} = K_{pj} - K_{wj} C$.

From Eqs. (10) and (11), we obtain a 2D model of the closed-loop augmented system

$$\dot{\varphi}(k,\tau)=\sum_{j=1}^{r}\sum_{i=1}^{r}\theta_i(\lambda(k,\tau))\theta_j(\lambda(k,\tau))[\tilde{A}\varphi(k,\tau)+\tilde{A}_d\varphi(k-1,\tau)+\tilde{B}v_w(k-1,\tau)] \quad (12)$$

where

$$\varphi(k,\tau)=\begin{bmatrix} x_p^{\mathrm{T}}(k,\tau) & x_q^{\mathrm{T}}(k,\tau)\end{bmatrix}^{\mathrm{T}},$$

$$\tilde{A}=\begin{bmatrix} A_i+B_i\tilde{K}_{pj} & B_iK_{qj} \\ -\omega_cC-CA_i-CB_i\tilde{K}_{pj} & -\omega_cI-CB_iK_{qj}\end{bmatrix},$$

$$\tilde{A}_d=\begin{bmatrix} 0 & 0 \\ 0 & \omega_cI\end{bmatrix},\ \tilde{B}=\begin{bmatrix} wB_iK_{wj} \\ -wCB_iK_{wj}\end{bmatrix}.$$

It should be noted that $v_w(k-1,\tau)$ is the learning variable, and $\varphi(k,\tau)$ is the control variable. Thus, tuning $\tilde{K}_{pj}$, K_{qj}, and K_{wj} allows us to achieve a trade-off between the control performance and stability.

3. Fuzzy MRC System Design

Lemma 1: (**Schur complement**) [22]: For a given symmetric matrix, there is

$$X=\begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22}\end{bmatrix}.$$

It is noted that the three conditions are easily confirmed and are equivalent to be the following:

(1) $X<0$;

(2) $X_{11}<0,\ X_{22}-X_{12}^{\mathrm{T}}X_{11}^{-1}X_{12}<0$;

(3) $X_{22}<0,\ X_{11}-X_{12}X_{22}^{-1}X_{12}^{\mathrm{T}}<0$.

Theorem 1: For a given ω_c and four positive scalars α, β, γ, and w, it is assumed that there are symmetrical and positive-definite matrices X_1, Y_1, X_2, and Y_2, and arbitrary matrices with appropriate dimensions W_{1i}, W_{2i}, and W_{3i} such that the LMIs hold for $1\le i\le j\le r$:

$$\Lambda_{ii}<0 \quad (13)$$

$$\Lambda_{ij}+\Lambda_{ji}<0 \quad (14)$$

where

$$\Lambda_{ij}=\begin{bmatrix}
\Lambda_{1,1}^{ij} & \Lambda_{1,2}^{ij} & 0 & 0 & \Lambda_{1,5}^{ij} & \alpha X_1 & 0 & \Lambda_{1,8}^{ij} \\
* & \Lambda_{2,2}^{ij} & 0 & \gamma\omega_cY_2 & \Lambda_{2,5}^{ij} & 0 & \beta X_2 & 0 \\
* & * & -Y_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma Y_2 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Lambda_{5,5}^{ij} & 0 & 0 & 0 \\
* & * & * & * & * & -Y_1 & 0 & 0 \\
* & * & * & * & * & * & -\gamma Y_2 & 0 \\
* & * & * & * & * & * & * & -\gamma Y_2
\end{bmatrix}$$

$$\Lambda_{1,1}^{ij}=\alpha(A_i+B_i\tilde{K}_{pj})X_1+\alpha X_1(A_i+B_i\tilde{K}_{pj})^{\mathrm{T}},$$
$$\Lambda_{1,2}^{ij}=\beta B_iK_{qj}X_2+\alpha X_1(-\omega_cC-CA_i-CB_i\tilde{K}_{pj})^{\mathrm{T}},$$
$$\Lambda_{1,5}^{ij}=\gamma wB_iK_{wj}Y_2-\alpha wX_1C^{\mathrm{T}},\ \Lambda_{1,8}^{ij}=-\alpha X_1C^{\mathrm{T}},$$
$$\Lambda_{2,2}^{ij}=-2\beta\omega_cX_2-\beta CB_iK_{qj}X_2-\beta X_2(CB_iK_{qj})^{\mathrm{T}},$$
$$\Lambda_{2,5}^{ij}=-\gamma wCB_iK_{wj}Y_2,\ \Lambda_{5,5}^{ij}=\gamma Y_2(w^2-1).$$

Then, the closed-loop augmented system is asymptotically stable. Moreover, the 2D fuzzy repetitive controller gains are given by $\tilde{K}_{pi}=W_{1i}X_1{}^{-1}$, $K_{qi}=W_{2i}X_2^{-1}$, $K_{wi}=W_{3i}Y_2^{-1}$, and $K_{pi}=W_{1i}X_1^{-1}+K_{wi}C$.

Remark 1: The structure of the developed system guarantees the tracking performance when the system is stable. $V_1(k,\tau)$, $V_2(k,\tau)$, and $V_3(k,\tau)$ are quadratic items concerned with the system control performance. The tuning parameters α, β, γ, and w in LMIs (13) and (14) influence the gains K_{pj}, K_{qj}, and K_{wj} to regulate the control performance. The specific effects of adjusting the tuning parameters are presented in Section 4.

Remark 2: The new modified repetitive controller has properties of both the repetitive controller and modified repetitive controller. The extra positive-feedback line in **Fig. 1** adds a relaxing variable W_{3j} to ensure a feasible solution of the LMIs. The variable w adjusts the weight between RC and MRC to achieve a trade-off between the tracking performance and stability. Moreover, when $w=0$, the developed repetitive controller is the same as a basic modified repetitive controller. Thus, it is easy to compare the effects of the two methods.

Proof: Consider a 2D Lyapunov functional candidate as

$$V(k,\tau)=V_1(k,\tau)+V_2(k,\tau)+V_3(k,\tau) \quad (15)$$

where

$$V_1(k,\tau)=\varphi^{\mathrm{T}}(k,\tau)P\varphi(k,\tau),$$
$$P=\mathrm{diag}\{\frac{1}{\alpha}P_1\ \ \frac{1}{\beta}P_2\},$$
$$V_2(k,\tau)=\int_{\tau-T}^{\tau}\varphi^{\mathrm{T}}(k,s)Q\varphi(k,s)ds,$$
$$Q=\mathrm{diag}\{Q_1\ \ \frac{1}{\gamma}Q_2\},$$
$$V_3(k,\tau)=\frac{1}{\gamma}v_w^{\mathrm{T}}(k,\tau)Q_2v_w(k,\tau).$$

The system is stable when the derivative of $V(k,\tau)$ is less than zero. P_1, Q_1, P_2, and Q_2 are positive-definite matrices to be determined. Let $P_1=X_1^{-1}$, $Q_1=Y_1^{-1}$, $P_2=X_2^{-1}$, and $Q_2=Y_2^{-1}$.

Along the closed-loop system (12), the following incremental functions are yielded.

$$\delta V(k,\tau)=\frac{dV_1(k,\tau)}{d\tau}+\frac{dV_2(k,\tau)}{d\tau}+\Delta V_3(k,\tau),$$

$$\begin{aligned}\dot{V}_1(k,\tau)&=2\varphi^{\mathrm{T}}(k,\tau)P\dot{\varphi}(k,\tau)\\&=2\varphi^{\mathrm{T}}(k,\tau)P\tilde{A}\varphi(k,\tau)\\&\quad+2\varphi^{\mathrm{T}}(k,\tau)P\tilde{A}_d\varphi(k-1,\tau)\\&\quad+2\varphi^{\mathrm{T}}(k,\tau)P\tilde{B}v_w(k-1,\tau),\end{aligned}$$

$$\begin{aligned}\dot{V}_2(k,\tau)&=\varphi^{\mathrm{T}}(k,\tau)Q\varphi(k,\tau)\\&\quad-\varphi^{\mathrm{T}}(k-1,\tau)Q\varphi(k-1,\tau),\end{aligned}$$

$$\begin{aligned}\Delta V_3(k,\tau) &= \frac{1}{\gamma}v_w^{\mathrm{T}}(k,\tau)Q_2v_w(k,\tau)\\&\quad -\frac{1}{\gamma}v_w^{\mathrm{T}}(k-1,\tau)Q_2v_w(k-1,\tau)\\&= \frac{1}{\gamma}\varphi^{\mathrm{T}}(k,\tau)\bar{C}^{\mathrm{T}}Q_2\bar{C}\varphi(k,\tau)\\&\quad +\frac{1}{\gamma}w\varphi^{\mathrm{T}}(k,\tau)\bar{C}^{\mathrm{T}}Q_2v_w(k-1,\tau)\\&\quad +\frac{1}{\gamma}wv_w{}^{\mathrm{T}}(k-1,\tau)Q_2\bar{C}\varphi(k,\tau)\\&\quad +\frac{1}{\gamma}(w^2-1)v_w{}^{\mathrm{T}}(k-1,\tau)Q_2v_w(k-1,\tau).\end{aligned}$$

The derivative of $V(k,\tau)$ is rewritten as

$$\delta V(k,\tau) = \sum_{j=1}^{r}\sum_{i=1}^{r}\theta_i(\lambda(k,\tau))\theta_j(\lambda(k,\tau)) \chi^{\mathrm{T}}(k,\tau)\Pi_{ij}\chi(k,\tau) \quad . . . (16)$$

where

$$\chi(k,\tau) = [\varphi^{\mathrm{T}}(k,\tau) \quad \varphi^{\mathrm{T}}(k-1,\tau) \quad v_w^{\mathrm{T}}(k-1,\tau)],$$

$$\Pi_{ij} = \begin{bmatrix} \Pi_{1,1}^{ij} & P\tilde{A}_d & P\tilde{B}+\frac{1}{\gamma}\bar{C}^{\mathrm{T}}Q_2w \\ * & -Q & 0 \\ * & * & \frac{1}{\gamma}Q_2(w^2-1) \end{bmatrix},$$

$$\Pi_{1,1}^{ij} = 2P\tilde{A}+\frac{1}{\gamma}\bar{C}^{\mathrm{T}}Q_2\bar{C}+Q.$$

The right-hand side of Eq. (16) is the same as

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(\lambda(k,\tau))\theta_j(\lambda(k,\tau))\chi^{\mathrm{T}}(k,\tau)\Pi_{ij}\chi(k,\tau)\\&=\sum_{i=1}^{r}\theta_i^2(\lambda(k,\tau))\chi^{\mathrm{T}}(k,\tau)\Pi_{ii}\chi(k,\tau)\\&\quad+\sum_{i=1}^{r}\sum_{i<j}\theta_i(\lambda(k,\tau))\theta_j(\lambda(k,\tau))\\&\qquad\chi^{\mathrm{T}}(k,\tau)(\Pi_{ij}+\Pi_{ji})\chi(k,\tau).\end{aligned} \quad (17)$$

When $\Pi_{ii} < 0$ and $\Pi_{ij}+\Pi_{ji} < 0$, $\dot{V}(k,\tau) < 0$ for $\chi(k,\tau) \neq 0$. Furthermore, according to Lemma 1, $\Pi_{ij} < 0$ is equivalent to the LMI as follows:

$$\Pi_{ij} = \begin{bmatrix} \Pi_{1,1}^{ij} & \Pi_{1,2}^{ij} & 0 & 0 & \Pi_{1,5}^{ij} & Q_1 & 0 & -C^{\mathrm{T}} \\ * & \Pi_{2,2}^{ij} & 0 & \Pi_{2,4}^{ij} & \Pi_{2,5}^{ij} & 0 & \Pi_{2,7}^{ij} & 0 \\ * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{4,4}^{ij} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{5,5}^{ij} & 0 & 0 & 0 \\ * & * & * & * & * & -Q_1 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{7,7}^{ij} & 0 \\ * & * & * & * & * & * & * & -\gamma Q_2^{-1} \end{bmatrix} < 0 \quad (18)$$

where

$$\begin{aligned}\Pi_{1,1}^{ij} &= \frac{1}{\alpha}P_1(A_i+B_i\tilde{K}_{pj})+\frac{1}{\alpha}(A_i+B_i\tilde{K}_{pj})^TP_1,\\ \Pi_{1,2}^{ij} &= \frac{1}{\alpha}P_1B_iK_{qj}-\frac{1}{\beta}\omega_cC^{\mathrm{T}}P_2\\&\quad -\frac{1}{\beta}A_i{}^{\mathrm{T}}C^{\mathrm{T}}P_2-\frac{1}{\beta}\tilde{K}_{pj}^{\mathrm{T}}B_i{}^{\mathrm{T}}C^{\mathrm{T}}P_2,\\ \Pi_{1,5}^{ij} &= \frac{1}{\alpha}wP_1B_iK_{wj}-\frac{1}{\gamma}wC^{\mathrm{T}}Q_2,\\ \Pi_{2,2}^{ij} &= -(2/\beta)\omega_cP_2-\frac{1}{\beta}P_2CB_iK_{qj}\\&\quad -\frac{1}{\beta}(CB_iK_{qj})^{\mathrm{T}}P_2,\\ \Pi_{2,4}^{ij} &= \frac{1}{\beta}P_2\omega_c,\ \Pi_{2,5}^{ij} = -\frac{1}{\beta}wP_2CB_iK_{wj},\\ \Pi_{2,7}^{ij} &= \frac{1}{\gamma}Q_2,\ \Pi_{4,4}^{ij} = -\frac{1}{\gamma}Q_2,\\ \Pi_{5,5}^{ij} &= \frac{1}{\gamma}(w^2-1)Q_2,\ \Pi_{7,7}^{ij} = -\frac{1}{\gamma}Q_2.\end{aligned}$$

Pre- and post-multiplying Eq. (18) by diag$\{\alpha X_1\ \beta X_2\ Y_1\ \gamma Y_2\ \gamma Y_2\ Y_1\ \gamma Y_2\ I\}$ yield Eqs. (13) and (14), respectively. Further, the controller gains are $\tilde{K}_{pi} = W_{1i}X_1{}^{-1}$, $K_{qi} = W_{2i}X_2^{-1}$, $K_{wi} = W_{3i}Y_2^{-1}$, and $K_{pi} = W_{1i}X_1^{-1}+K_{wi}C$. ■

4. Numerical Example

Consider the voltage control of Chua' circuit systems [23, 24].

$$\begin{aligned}\dot{x}_{p1}(t) &= -\sigma_1x_{p1}(t)+\sigma_1x_{p2}(t)-\sigma_1f(x_{p1}(t))+u_1(t)\\ \dot{x}_{p2}(t) &= x_{p1}(t)-x_{p2}(t)+x_{p3}(t)+u_2(t)\\ \dot{x}_{p3}(t) &= -\sigma_2x_{p2}(t)+u_3(t)\\ y_p(t) &= x_{p1}(t)\end{aligned} \quad (19)$$

where $\sigma_1 = 10$, $\sigma_2 = 14.87$, and $d = 1.8$. $x_{p1}(t)$, $x_{p2}(t)$, and $x_{p3}(t)$ are the two voltages and current of the Chua's circuit, respectively. $f(x_{p1}(t)) = g_bx_{p1}(t)+0.5(g_a-g_b)(|-|x_{p1}(t)-1|+x_{p1}(t)+1|)$, where $g_a = -1.27$ and $g_b = -0.68$, is nonlinear term.

The dynamic systems (19) is represented by the T–S fuzzy model with two fuzzy rules.

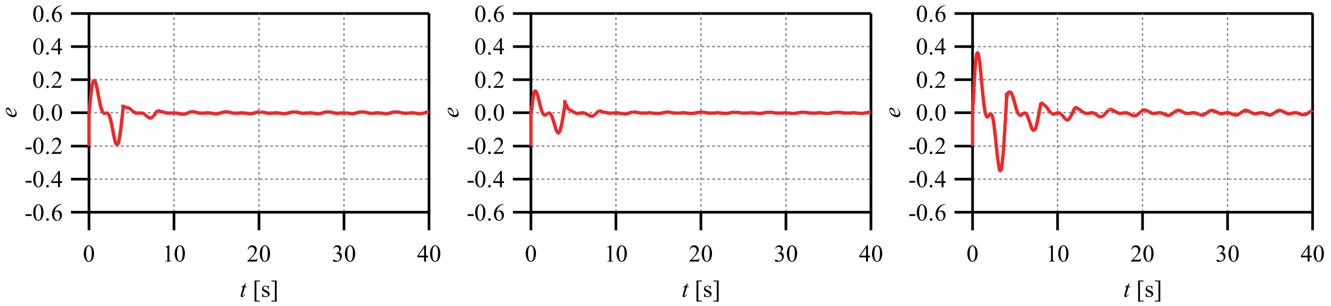
R^1: If $x_{p1}(t)$ is $F_1(x_{p1}(t))$, then

$$\begin{aligned}\dot{x}_p(t) &= A_1x_p(t)+B_1u(t),\\ y_p(t) &= Cx_p(t).\end{aligned} \quad . . . . (20)$$

R^2: If $x_{p1}(t)$ is $F_2(x_{p1}(t))$, then

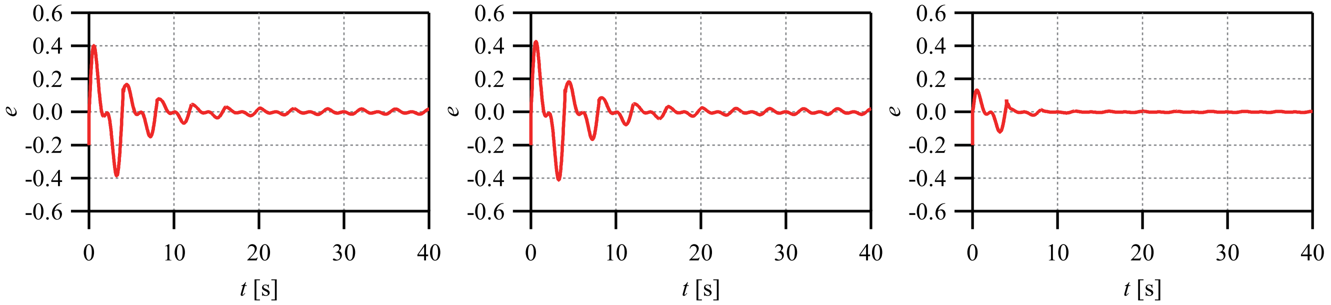
$$\begin{aligned}\dot{x}_p(t) &= A_2x_p(t)+B_2u(t),\\ y_p(t) &= Cx_p(t).\end{aligned} \quad . . . . (21)$$

The membership functions of the fuzzy sets are then



(a) $\alpha = 40$, $\beta = 0.001$, $\gamma = 1$, $w = 0$ (b) $\alpha = 40$, $\beta = 0.001$, $\gamma = 1$, $w = 0.2$ (c) $\alpha = 40$, $\beta = 0.001$, $\gamma = 1$, $w = 0.7$

Fig. 2. Tracking error curves for the additional variable w.



(a) $\alpha = 30$, $\beta = 0.01$, $\gamma = 0.1$, $w = 0.2$ (b) $\alpha = 40$, $\beta = 0.01$, $\gamma = 1$, $w = 0.2$ (c) $\alpha = 40$, $\beta = 0.001$, $\gamma = 1$, $w = 0.2$

Fig. 3. Tracking error curves for the scalars α, β, γ, and w.

expressed as

$$\mu_1(x_{p1}) = \begin{cases} \frac{1}{2}\left(1 - \frac{f(x_{p1}(t))}{dx_{p1}(t)}\right), & x_{p1}(t) \neq 0 \\ \frac{1}{2}\left(1 - \frac{g_a}{d}\right), & x_{p1}(t) = 0 \end{cases}$$

$$\mu_2(x_{p1}) = 1 - \mu_1(x_{p1}).$$

where $u(t) = [u_1(t)\ u_2(t)\ u_3(t)]^{\mathrm{T}}$ is the control input. $x_p(t) = [x_{p1}(t)\ x_{p2}(t)\ x_{p3}(t)]^{\mathrm{T}}$ is the state variable. $x_{p1}(t)$ is the premise vector. The system matrices are given as follows:

$$A_1 = \begin{bmatrix} \sigma_1(d-1) & \sigma_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\sigma_2 & 0 \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -\sigma_1(d+1) & \sigma_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\sigma_2 & 0 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Let us consider the tracking problem of a periodic reference input for a nonlinear system. The given periodic input is $r(t) = 0.5\sin(\pi t) + \sin(0.5\pi t)$. In order to track the periodic references as completely as possible, we set $T = 4$ s and $\omega_c = 100$ rad/s.

For the purpose of stabilizing the developed system, the search range for the tuning parameter w is selected as $w \in [0, 1)$. The variable w achieves a trade-off between the tracking performance and stability. As shown in **Fig. 2**, the system almost always enters a stable state after about two periods for different variables w. Therefore, it influences the learning rather than the control. The variable w has a primary effect on the tracking performance as compared to other variables. Thus, $w = 0.2$ is first selected to achieve a good control performance.

As mentioned in Remark 1, tuning α, β, γ, and w in LMIs (13) and (14) influences the gains K_{pj}, K_{qj}, and K_{wj} to regulate the control and learning preferentially. It is noted that tuning β and w influences K_{qj} and K_{wj}. The system exhibits a significant improvement as tuning β and w is related to a greater extent with the control performance than the other parameters. The tracking error in **Fig. 3(c)** at the first period is much smaller than that in **Figs. 3(a)** and **(b)**. Thus, regulating β mainly affects the control behavior. Some of the obtained results are shown in **Fig. 3**.

The determined parameters are then finally selected as shown below:

$$\alpha = 40,\ \beta = 0.001,\ \gamma = 1.$$

Subsequent to solving the LMIs shown in Theorem 1, we obtain a feasible solution. Then, it is easy to obtain the resulting feedback controller gains:

$$K_{p1} = \begin{bmatrix} -26.0726 & -5.0199 & 0.0000 \\ -126.3846 & -0.3598 & 6.9350 \\ 0.0000 & 6.9350 & -1.3598 \end{bmatrix},$$

$$K_{p2} = \begin{bmatrix} 9.9274 & -5.0199 & 0.0000 \\ -126.3846 & -0.3598 & 6.9350 \\ 0.0000 & 6.9350 & -1.3598 \end{bmatrix},$$

$$K_{q1} = \begin{bmatrix} 287.7037 \\ 477.1813 \\ 0.0000 \end{bmatrix}, K_{q2} = \begin{bmatrix} 287.7037 \\ 477.1813 \\ 0.0000 \end{bmatrix},$$

$$K_{w1} = \begin{bmatrix} 30.0472 \\ 0.0000 \\ 0.0000 \end{bmatrix}, K_{w2} = \begin{bmatrix} 30.0472 \\ 0.0000 \\ 0.0000 \end{bmatrix}.$$

The initial state of the system for different parameters is $x_p(0) = [0.2\ 0.5\ 1]^{\mathrm{T}}$. Thus, all the initial tracking errors are 0.2. They are presented in **Figs. 2** and **3**.

The simulation results show that the constant w has an optimal value when the system has a good dynamic response. As mentioned in Remark 2, when w is selected to be an appropriate value or change in a certain interval, the developed system achieves a good tracking performance. The adjustment of the other parameters is related to a lesser extent with the control performance. Therefore, the complexity of the parameter adjustment is reduced. Moreover, when w is zero, the developed repetitive controller is a modified repetitive controller. The comparison shown in **Figs. 2(a)** and **(b)** demonstrates that the developed method has a better tracking performance.

Figure 4 displays the periodic reference signal and the output of the fuzzy system, which shows that the system has a good tracking performance. The output tracks the reference quickly after approximately two periods. **Fig. 5** exposes the states of the fuzzy system. It shows that the system quickly enters the steady state. All the simulation results verify that the control performance of the developed method is satisfactory.

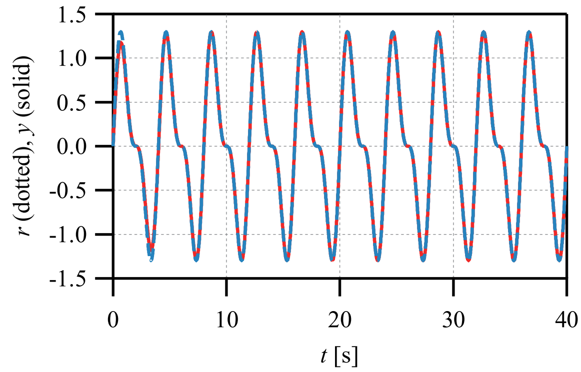


Fig. 4. Periodic tracking reference $r(t)$ and system output $y(t)$.

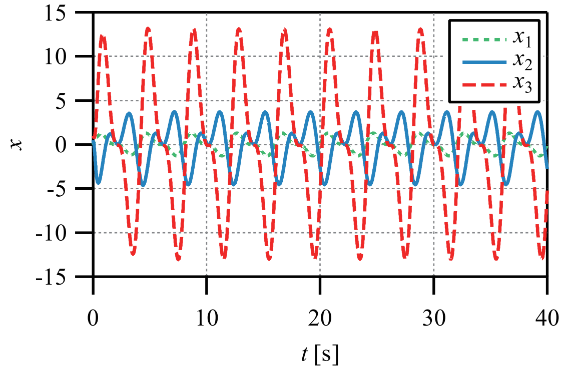


Fig. 5. System states $x_1(t)$, $x_2(t)$, and $x_3(t)$.

5. Conclusion

This paper presents a new MRC method that exhibits a balance between the control performance and stability for a class of affine nonlinear systems. First, the T–S fuzzy model was used to transform a nonlinear system into linear subsystems. The methods for a linear RC system can be simply extended to a nonlinear RC system. By exploiting the 2D nature of an RC process, a 2D representation of the system was established, and the 2D control law regulated the control and learning preferentially. A sufficient condition in terms of LMIs was then obtained to design the PDC-based state-feedback controllers using the Lyapunov stability theory and 2D system theory. Finally, simulations and a comparison show that the new method provides a better periodic reference tracking performance than the conventional MRC method, and the developed system has the best tracking performance when the constant w appropriately takes a value from 0 to 1.

The method of obtaining the exact value of w will be discussed further in the near future. In addition, the independent adjustment of the control and learning is also an interesting topic for potential future research.

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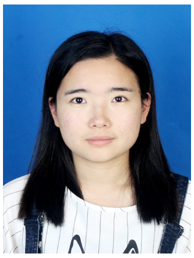

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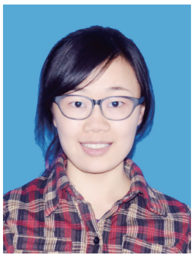

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