https://doi.org/10.20965/jaciii.2016.p1119

Paper:

Block Sparse Signal Reconstruction Using Block-Sparse Adaptive Filtering Algorithms

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[Received July 6, 2016; accepted September 29, 2016]

Sparse signal reconstruction (SSR) problems based on compressive sensing (CS) arise in a broad range of application fields. Among these are the so-called "block-structured" or "block sparse" signals with nonzero atoms occurring in clusters that occur frequently in natural signals. To make block-structured sparsity use more explicit, many block-structure-based SSR algorithms, such as convex optimization and greedy pursuit, have been developed. Convex optimization algorithms usually pose a heavy computational burden, while greedy pursuit algorithms are overly sensitive to ambient interferences, so these two types of block-structure-based SSR algorithms may not be suited for solving large-scale problems in strong interference scenarios. Sparse adaptive filtering algorithms have recently been shown to solve large-scale CS problems effectively for conventional vector sparse signals. Encouraged by these facts, we propose two novel block-structure-based sparse adaptive filtering algorithms, i.e., the "block zero attracting least mean square" (BZA-LMS) algorithm and the "block ℓ_0-norm LMS" (BL0-LMS) algorithm, to exploit their potential performance gain. Experimental results presented demonstrate the validity and applicability of these proposed algorithms.

Keywords: compressive sensing, sparse signal reconstruction, block-structured sparsity, least mean square, sparse constraint

1. Introduction

Compressive sensing (CS) theory has been applied in such engineering fields as signal processing, wireless communications, and large-scale system analysis [1, 2]. One of the main issues in CS theory is how to reconstruct sparse signals via a compressive sampling method [3], i.e., original sparse signal $\boldsymbol{s}$ in N-dimension domain with K nonzero atoms ($K \ll N$) can be pre-transformed to a down-sampling signal $\boldsymbol{y}$ in M-dimension domain ($K \leq M \ll N$) utilizing a suitable sensing matrix $\boldsymbol{A}$ ($M \times N$) in transmitter, which yields the following underdetermined linear equation

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s}. \quad (1)$$

In the receiver after transmission, $\boldsymbol{s}$ is reconstructed effectively from underdetermined Eq. (1) using sparse signal reconstruction (SSR) algorithms.

In the sections that follow, we present a type of sparse signal with a special structure termed a block sparse signal, i.e., nonzero atoms in the original signal exist as clusters rather than being spread arbitrarily throughout the unknown signal [4]. The difference between conventional vector sparse signals and block sparse signals is shown visibly in **Fig. 1**. Specifically, original block sparse signal $\boldsymbol{s}$ can be expressed as below,

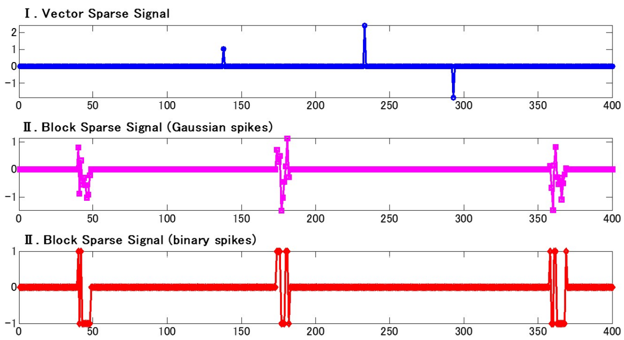

Fig. 1. Vector sparse signal vs. block sparse signals.

$$\boldsymbol{s} = \left[\underbrace{s_1,\ldots,s_d}_{\boldsymbol{s}^T[1]},\underbrace{s_{d+1},\ldots,s_{2d}}_{\boldsymbol{s}^T[2]},\ldots,\underbrace{s_{N-d+1},\ldots,s_N}_{\boldsymbol{s}^T[m]}\right]^T, \quad (2)$$

$N = m \cdot d$, d denotes the blocked length, and m denotes the number of separated blocks. Based on Eq. (2), $\boldsymbol{s}$ is termed block K-sparse where $K \in \{1, 2, \ldots, m\}$, when $\boldsymbol{s}$ has at most K blocks involving nonzero atoms. It is formulized in Eq. (3) [4],

$$K \geq \|\boldsymbol{s}\|_{2,0} = \sum_{i=1}^{m} I\left(\|\boldsymbol{s}[i]\|_2 > 0\right), \quad (3)$$

where $I(\cdot)$ denotes indicator function [4], defined as

$$I(\|\boldsymbol{s}[i]\|_2 > 0) = \begin{cases} 1, & \|\boldsymbol{s}[i]\|_2 > 0 \\ 0, & \text{elsewhere} \end{cases} \quad \ldots \quad (4)$$

This special type of sparse signal usually arises in practical applications of multi-band signals [5–8] or of gene expression level measurement [9]. Network topology identification [10, 11], source localization in sensor networks [12, 13], MIMO channel equalization [12, 13], using block sparsity gives us better reconstruction properties than simply treating signals as sparse in the conventional sense [4].

Algorithms in block versions proposed for implementing block sparse signal reconstruction fall mainly into two basic classes, i.e., convex relaxation and greedy pursuit. For the convex relaxation class, the basis pursuit (BP) algorithm based on linear programming (LP) has been properly generalized into a mixed ℓ_2/ℓ_1-norm minimization recovery algorithm [4, 14]. For the greedy pursuit class, extension versions of the compressive sampling matching pursuit (CoSaMP) algorithm and the iterative hard thresholding (IHT) algorithm for block sparse signal reconstruction have been presented in [12]. Block versions of the matching pursuit (MP), orthogonal MP (OMP) [15], and stagewise OMP (StOMP) algorithms [16] are termed BMP, BOMP [4], and BStOMP [17].

The BStOMP algorithm in particular obtains excellent reconstruction performance when the ambient noise level is moderate. As pointed out in [17, 18], however, convex optimization algorithms generally require a high computational burden due to their high complexity, preventing them from being practicable in large-scale applications. For greedy pursuit algorithms, deteriorating reconstruction performance under strong noise interference is considerable.

These unsuitable scenarios inspired us to propose more practical, efficient block versions of SSR algorithms to solve block sparse signals reconstruction problems. Our purpose here is thus to clearly improve performance in a variety of scenarios.

Stochastic gradient-based sparse adaptive filtering algorithms have proven to be effective SSR algorithms [18], e.g., the ℓ_0-norm least mean square (ℓ_0-LMS) algorithm and the ℓ_0-norm exponentially forgetting window LMS (ℓ_0-EFWLMS) algorithm. These algorithms provide moderate computational complexity and improved robustness against ambient strong interference. Specifically, their reconstruction performance is superior to other classes of algorithms. State-of-the-art sparse adaptive filtering algorithms reconstruct vector sparse signals in the conventional sense, without accounting for more general cases for block sparse signals. We therefore extend conventional sparse adaptive filtering algorithms to block versions, proposing two novel algorithms – the block zero attracting LMS (BZA-LMS) and the block ℓ_0-norm LMS (BL0-LMS) – both of which have the inherent advantages of conventional sparse adaptive filtering algorithms also sense and exploit the potential levels of sparsity of block-structured sparse signals, thus improving performance.

Table 1. Corresponding variables between CS problem and adaptive framework.

CS Problem	Adaptive Framework
$\boldsymbol{a}_j, j \in \{1,2,\ldots,M\}$	$\boldsymbol{x}^T(n)$
$\boldsymbol{s}(n)$	$\boldsymbol{h}(n)$
$y_j = \boldsymbol{a}_j\boldsymbol{s} + v_j$	$d(n) = \boldsymbol{x}^T(n)\boldsymbol{h} + z(n)$

Section 2 of this paper reviews conventional sparse adaptive filtering algorithms. Sections 3 systematically describes the proposed algorithms and Section 4 provides simulation results in different scenarios. Section 5 lists our conclusions.

2. Sparse Adaptive Filtering Algorithms: A Review

This section reviews the SSR method based on an adaptive filtering framework and the sparse constraint adopted in recursion updates.

2.1. Adaptive Filtering Framework for Sparse Signal Reconstruction

Based on the CS problem, we reconstruct sparse signal $\boldsymbol{s}$ from the underdetermined Eq. (1). Suppose that

$$\boldsymbol{s} = [s_1, s_2, \ldots, s_N]^T, \quad \ldots \quad (5)$$

$$\boldsymbol{a}_j = [a_{j1}, a_{j2}, \ldots, a_{jN}],\ j \in \{1,2,\ldots,M\}, \quad \ldots \quad (6)$$

$$\boldsymbol{A} = [\boldsymbol{a}_1^T, \boldsymbol{a}_2^T, \ldots, \boldsymbol{a}_M^T]^T, \quad \ldots \quad (7)$$

$$\boldsymbol{y} = [y_1, y_2, \ldots, y_M]^T, \quad \ldots \quad (8)$$

$$\boldsymbol{v} = [v_1, v_2, \ldots, v_M]^T. \quad \ldots \quad (9)$$

Given that in realistic transmission, harmful ambient additive interference is unavoidable, we take Gaussian distribution-based noise interference $\boldsymbol{v}$ into consideration. The updated underdetermined equation is as follows:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{v}. \quad \ldots \quad (10)$$

Adaptive filtering algorithms [19] are fairly practical algorithms with simple structures, a certain noise cancelation capability, and outstanding signal reconstruction performance. Recursion error constructing the cost function of adaptive algorithms is as follows:

$$e(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{h}(n), \quad \ldots \quad (11)$$

$d(n)$ denotes the desired signal contaminated by additive noise $z(n)$, $\boldsymbol{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^T$ and $\boldsymbol{h}(n) = [h_1(n), h_2(n), \ldots, h_N(n)]^T$ denote the input signal and the iterative reconstruction signal. The iteration run is denoted by n. Through iteratively minimizing $e(n)$, $\boldsymbol{h}(n)$ is reconstructed increasingly accurately. The CS problem was solved in the adaptive framework based on the corresponding variables listed in **Table 1** [18], row vectors $\boldsymbol{a}_j$ in sensing matrix $\boldsymbol{A}$ are used as training sequences

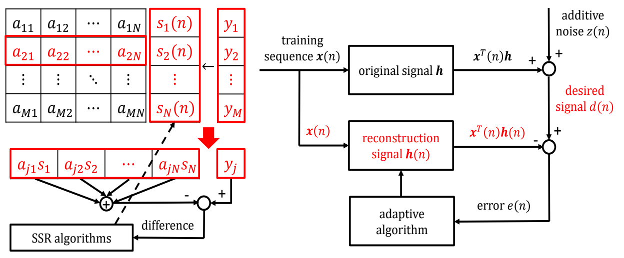

Fig. 2. Solve CS problem by adaptive framework.

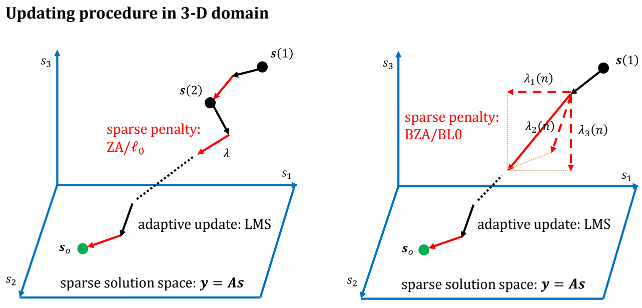

Fig. 3. Updating procedures between conventional sparse vs. block-sparse adaptive filtering algorithms.

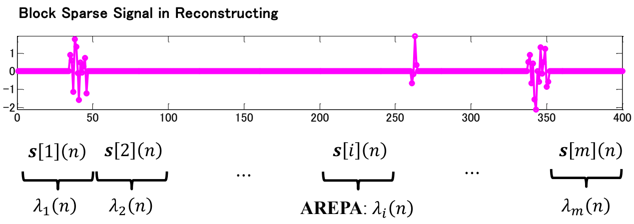

Fig. 4. Adaptive regularization parameter series.

Table 2. Adaptive filtering framework.

i. Initialize $\boldsymbol{s}(0) = \boldsymbol{0}$, $n = 1$;

ii. Input data $\boldsymbol{a}_j$ and y_j to adaptive filter, where

$$j = \bmod(n, M), \quad \ldots \ldots \ldots \quad (12)$$

the operator $\bmod(\bullet)$ denotes modulo operation which finds the remainder after division;

iii. Impose adaptive algorithms to reconstruct $\boldsymbol{s}(n)$;

iv. Termination condition justification,

$$\|\boldsymbol{s}(n) - \boldsymbol{s}(n-1)\|_2 < \varepsilon \ \text{or}\ n > C, \quad . \quad (13)$$

where positive number ε is a given error tolerance and C denotes the maximum number of iteration runs;

v. When termination condition is unsatisfied, n increases by one then return to Step *ii.* to continue recursion loop; Otherwise, output $\boldsymbol{s}(n)$ and exit.

$\boldsymbol{x}^T(n)$, and each component y_j of down-sampling signal is viewed as obtained desired signal $d(n)$, in the reconstruction process, $\boldsymbol{a}_j$ and y_j are used circularly. The CS problem is solved by using an adaptive framework as shown in **Fig. 2**. The specific reconstruction procedure is shown in **Table 2**.

2.2. Sparse Constraint upon Recursion Update

Note that standard adaptive filtering algorithms cannot obtain sufficient performance gain because no use is made of signal sparsity. In fact, many effective sparse adaptive filtering algorithms based on the widely used LMS algorithm [18, 20–25] involve sparse constraint restriction in their cost function, e.g., typical convex zero attracting and the optimal nonconvex ℓ_0-norm. Specifically, in [18], the ℓ_0-LMS algorithm has been shown to stably and markedly improve performance in solving the CS problem. The updating of the ZA-LMS and ℓ_0-LMS algorithms is shown at left in **Fig. 3**.

3. Proposed Block-Sparse Algorithms

Conventional sparse adaptive filtering algorithms focus on reconstructing vector sparse signals and thus, in each iteration, implement uniform or reweighted sparse penalties for all components of recursion updated reconstruction signal $\boldsymbol{s}(n)$. However, in the case for block sparse signals reconstruction, the differences of sparsity degree in different segments of original signal are fairly evident, which means that the block-structured sparsity information may not be sufficiently exploited, making it necessary to impose characterized sparse adaptive filtering algorithms to effectively exploit block-structured sparsity.

3.1. Adaptive Regularization Parameter Series

Regularization parameter (REPA) λ is a quite critical parameter, which plays an important role in balancing recursion update term and sparsity exploitation term [26]. Adaptive REPA (AREPA) parameter λ_N is applied more effectively in scenarios as shown in [27]. In our paper, two novel block versions of algorithms are proposed, i.e., the BZA-LMS algorithm and the BL0-LMS algorithm, which introduce a series of AREPA $\lambda_i(n)$, $i \in \{1, 2, \ldots, m\}$ in their cost function, to more accurately sense sparsity information via blocked transaction for original block sparse signals, as shown in **Fig. 4**. The next 2 sections detail a description of the proposed algorithms.

3.2. BZA-LMS Algorithm

We define the BZA-LMS algorithm cost function as

$$G_{ZA}(n) = e^2(n) + \lambda_i(n) \|\boldsymbol{s}[i](n)\|_1, \quad \ldots \ldots (14)$$

where $i \in \{1, 2, \ldots, m\}$. In Eq. (14), each AREPA is responsible for adaptively regularizing sparse penalty strength for each block of reconstruction signals, where the AREPA series formula is defined as

$$\lambda_i(n) = \lambda \frac{\left(\delta + \|\boldsymbol{s}[i](n)\|_2^2\right) \cdot \sigma^2 \cdot d}{\|\boldsymbol{s}[i](n)\|_2^2}, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

λ is redefined as an initial REPA parameter and adaptive regulation for the AREPA series is determined by the following four variables:

- $\|\boldsymbol{s}[i](n)\|_2^2$: Average power of reconstruction signal in blocks. This plays an important role in adaptive regulation against sparsity levels, which is the inverse ratio to AREPA, appropriately reducing sparse penalty strength when blocked signal power becomes high so that no redundant sparse penalty is applied.
- δ : Threshold of AREPA. This guarantees the application stability of AREPA.
- σ^2 : Variance of noise interferences. AREPA regulates its value adaptively based on different noise levels by using σ^2. This is a direct ratio to AREPA, appropriately increasing sparse penalty strength when noise strength is strong, improving the weight of sparsity exploitation.
- d : Blocked length. This is a direct ratio to AREPA. Assuming that the number of separated blocks increases, then $\|\boldsymbol{s}[i](n)\|_2^2$ will decrease correspondingly, resulting in an unexpected decrease of AREPA. To offset the decrease in sparse penalty strength, d is imported into the AREPA series.

The update recursion equation derived from the cost function Eq. (14) is as follows:

$$\boldsymbol{s}(n+1) = \boldsymbol{s}(n) + \mu e(n)\boldsymbol{x}(n) - \gamma_i(n)\,\mathrm{sgn}\left(\boldsymbol{s}[i](n)\right), \quad (16)$$

$\gamma_i(n) = \mu\lambda_i(n)$ denotes an adaptive zero attraction series. The specific reconstruction procedure for the BZA-LMS algorithm is shown in **Table 3**.

3.3. BL0-LMS Algorithm

Similar to the proposed BZA-LMS algorithm, the BL0-LMS algorithm involving the AREPA series produces an excellent effect when combined with the optimal ℓ_0-norm sparse constraint. ℓ_0-norm is a non-deterministic polynomial time (NP) hard problem, however, so here it is approximated by a continuous function [28]. The initial cost function of the BL0-LMS algorithm is defined as

$$G_{\ell_0}(n) = e^2(n) + \lambda_i(n)\|\boldsymbol{s}[i](n)\|_0, \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The advantages of the widely used approximation method in [28] stand out, e.g., in computational simplicity and robustness. The new cost function after ℓ_0-norm approximation is as follows:

$$G_{\ell_0}(n) = e^2(n) + \lambda_i(n) \sum_{l=1}^{d} \left(1 - \mathrm{e}^{-\alpha|s_l[i](n)|}\right), \quad . \quad (18)$$

Table 3. BZA-LMS algorithm.

i. Initialize $\boldsymbol{s}(0) = 0$, $n = 1$, set suitable μ, λ, δ, d by trial and error method;
ii. While termination condition Eq. (13) is unsatisfied;
iii. Select training sequence $x(n)$, and desired signal $d(n)$ contaminated by additive noise $z(n)$, $j = \bmod(n,M), \boldsymbol{x}^T(n) = \boldsymbol{a}_j, d(n) = y_j, z(n) = v_j$;
iv. Calculate recursion error, $e(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{s}(n)$;
v. Recursion update by LMS, $\boldsymbol{s}(n+1) = \boldsymbol{s}(n) + \mu e(n)\boldsymbol{x}(n)$;
vi. Design AREPA series $\lambda_i(n)$, and zero attraction parameter $\gamma_i(n)$, $\lambda_i(n) = \lambda \frac{\left(\delta + \|\boldsymbol{s}[i](n)\|_2^2\right) \cdot \sigma^2 \cdot d}{\|\boldsymbol{s}[i](n)\|_2^2}$, $\gamma_i(n) = \mu\lambda_i(n)$;
vii. Implement sparse penalty by BZA, $\boldsymbol{s}(n+1) = \boldsymbol{s}(n+1) - \gamma_i(n)\,\mathrm{sgn}(\boldsymbol{s}[i](n))$;
viii. The number of iteration increases by one, $n = n+1$;
ix. End while.

By minimizing Eq. (18), the corresponding recursion update equation is derived as

$$\begin{aligned} s_l[i](n+1) &= s_l[i](n) + \mu e(n)x[i](n+1-l) \\ &\quad - \gamma_i(n)\,\alpha\,\mathrm{sgn}\left(s_l[i](n)\right)\mathrm{e}^{-\alpha|s_l[i](n)|}, \end{aligned} \quad (19)$$

where $l \in \{1,2,\ldots,d\}$. To decrease the computational complexity of Eq. (19), which mainly comes from the sparse penalty term, first-order Taylor series expansion of exponential function is replaced by

$$\mathrm{e}^{-\alpha|s_l[i](n)|} \approx \begin{cases} 1 - \alpha\,|s_l[i](n)|, & |s_l[i](n)| \le \dfrac{1}{\alpha} \\ 0, & \text{elsewhere} \end{cases} \quad . \quad (20)$$

To simplify Eq. (19), we impose the following approximation equation

$$g\left(s_l[i](n)\right) = \begin{cases} \alpha + \alpha^2 s_l[i](n), & s_l[i](n) \in \left[-\dfrac{1}{\alpha}, 0\right) \\ \alpha - \alpha^2 s_l[i](n), & s_l[i](n) \in \left(0, \dfrac{1}{\alpha}\right] \\ 0, & \text{elsewhere} \end{cases}, \quad . \quad (21)$$

and the final update recursion equation becomes

$$\begin{aligned} s_l[i](n+1) &= s_l[i](n) + \mu e(n)x[i](n+1-l) \\ &\quad - \gamma_i(n)\,g\left(s_l[i](n)\right). \quad . \quad . \quad . \quad . \quad . \end{aligned} \quad (22)$$

The BL0-LMS algorithm is specifically reconstructed similar to **Table 3** contents, except for implementing stronger ℓ_0-norm sparse constraint strength. The BZA-LMS and BL0-LMS algorithms are updated as shown at right in **Fig. 3**. The sparse constraint strength of the ℓ_0-norm is much beyond the zero attracting ℓ_1-norm, so the BL0-LMS algorithm outperforms the BZA-LMS algorithm in SSR accuracy, as shown in simulation results below.

4. Simulation Results

We evaluated signal reconstruction performance and robustness against ambient interference by mean square deviation (MSD), and measured computational complexity by running time. For global parameters, block sparse signal $\boldsymbol{s}$ is modeled according to [17]: signal length is $N = 400$, down-sampling dimension is $M = 100$, the size of nonzero atoms in $\boldsymbol{s}$ is $S \in \{20, 40, 60\}$, block sparsity is $K \in \{2, 4, 6\}$ accordingly, and the location distribution of nonzero blocks and nonzero coefficients within each block yields discrete uniform. The magnitude of nonzero coefficients yields standard Gaussian distribution $\mathscr{CN}(0,1)$, each entry of sensing matrix $\boldsymbol{A}$ is independently generated from Gaussian distribution $\mathscr{N}(0, 1/M)$. Note that $\boldsymbol{s}$ is normalized in our experiments, and additive noise $\boldsymbol{v}$ is simulated by Gaussian noise s.t. $\mathscr{N}(0, 1/\sigma^2)$, where standard deviation is $\sigma \in \{5.5 \times 10^{-3}, 0.9 \times 10^{-2}, \ldots, 9.5 \times 10^{-2}\}$, and the signal-to-noise ratio (SNR) is defined as $10\log_{10}(\|\boldsymbol{As}\|_2^2/\sigma^2)$, so the SNR is set as $\{0\text{ dB}, 5\text{ dB}, \ldots, 25\text{ dB}\}$ in experiments. Monte-Carlo trials are set at 1000 times. For private parameters, step-size μ is set as 0.05, tolerance error $\varepsilon = 1 \times 10^{-4}$, and iteration upper limit $C = 1 \times 10^6$ for all sparse and block-sparse adaptive algorithms. The approximation equation parameter is $\alpha = 10$ for ℓ_0-LMS and BL0-LMS algorithms. For the proposed two block-sparse algorithms BZA-LMS and BL0-LMS, threshold δ of AREPA is set to 0.8, and the blocked length is designed as 20.

In the three experiments that follow, reference algorithms are chosen as greedy pursuit algorithms and their MSD results, generated in each stage within an inner loop, are shown. Exp. (1) verifies that greedy pursuit algorithms reach steady-state within 20 iteration runs. Experiment results show marked performance improvement using our two proposed algorithms, i.e., BZA-LMS and BL0-LMS. MSD performance comparisons are versus different block sparsity and noise interference levels in Experiments (1) and (2), and rigorous algorithms complexity is measured in Exp. (3).

Experiment (1): Reconstruction performance of our two proposed block-sparse adaptive filtering algorithms BZA-LMS and BL0-LMS, are verified and compared to four greedy pursuit algorithms under different block sparsity. In **Fig. 5**, the total number of nonzero components $S = 20$, the number of blocks of nonzero components $K = 2$ in original signal, SNR in communication environment is set as 15 dB. It is evident to find that sparse adaptive filtering algorithms ZA-LMS and ℓ_0-LMS much outperform greedy pursuit algorithms in reconstruction accuracy, which is consistent with the conclusion in [18]. Based on the effective use of block-structured sparsity, the MSD performance of the proposed algorithms is further

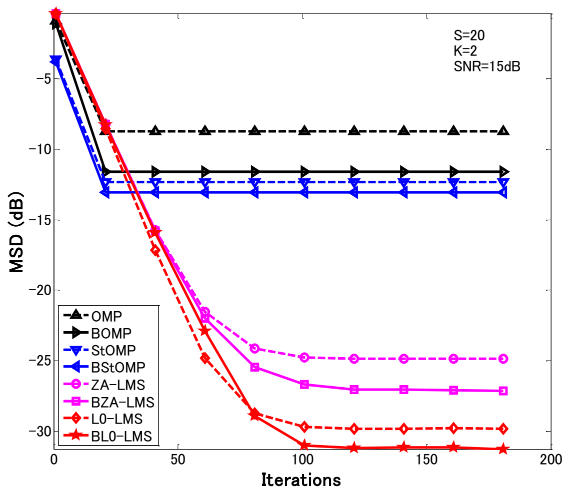

Fig. 5. MSD comparisons vs. block sparsity ($S = 20$, $K = 2$).

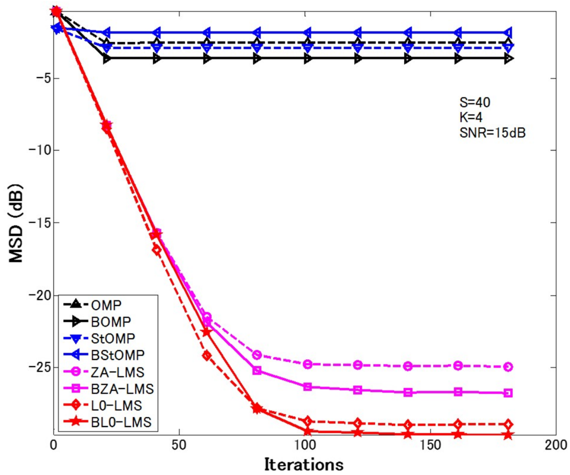

Fig. 6. MSD comparisons vs. block sparsity ($S = 40$, $K = 4$).

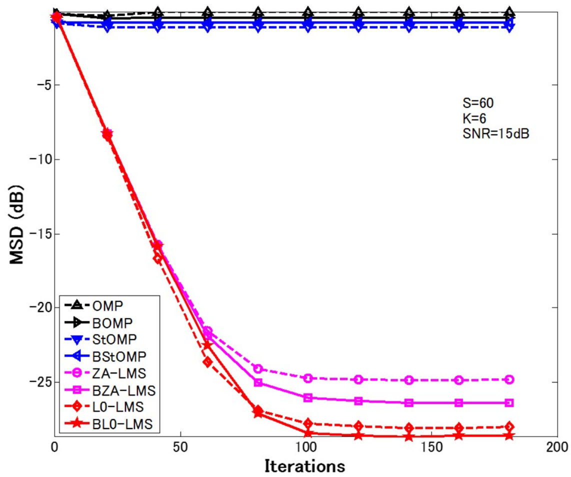

Fig. 7. MSD comparisons vs. block sparsity ($S = 60$, $K = 6$).

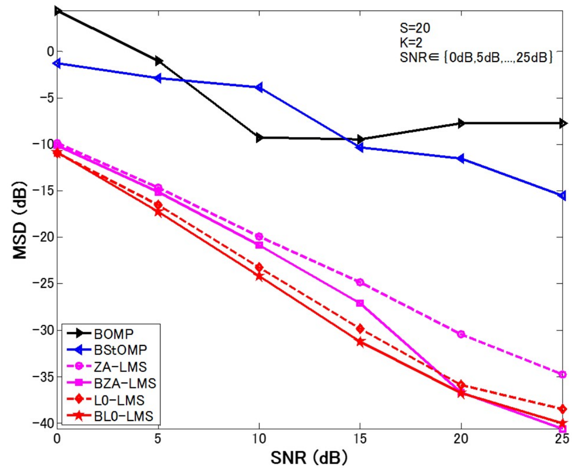

Fig. 8. Reconstruction MSD vs. SNR.

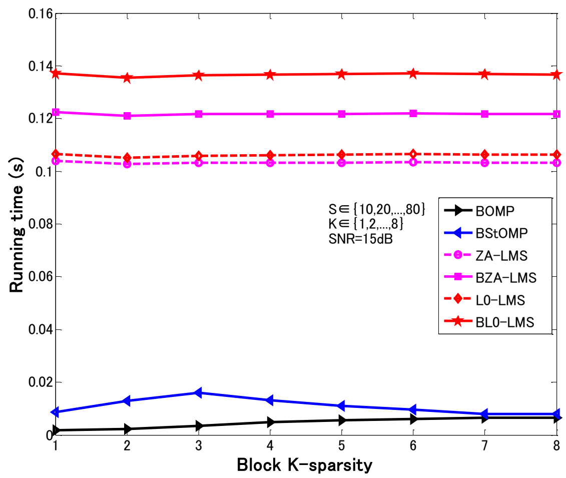

Fig. 9. Running time comparisons (unit: sec.).

clearly improved without sacrificing convergence iterations. In **Figs. 6** and **7**, block-sparsity is extended higher, i.e., $S = 40, K = 4$, and $S = 60, K = 6$, respectively, one can find that reference algorithms can hardly obtain contributive MSD performances. In contrast, sparse adaptive filtering algorithms ZA-LMS and ℓ_0-LMS exhibit superior performance in high block-sparsity scenarios, and the proposed BZA-LMS and BL0-LMS algorithms obtain even further performance gains.

Experiment (2): Compared to other SSR algorithm classes, the reconstruction stability of sparse adaptive filtering algorithms stands out under strong interference. In this experiment, we focus on the robustness property of both the proposed block versions of sparse adaptive algorithms, where the SNR bound is extended to $\{0\text{ dB}, 5\text{ dB}, \ldots, 25\text{ dB}\}$ in **Fig. 8**. When SNR increases from 15 dB to 20 dB–25 dB, conventional sparse adaptive algorithms ZA-LMS and ℓ_0-LMS show superior performance properties, and the proposed BZA-LMS and BL0-LMS algorithms obtain higher reconstruction accuracy. When ambient noise interference increases, however, SNR results decrease to 0 dB–10 dB, and the performance of block versions of greedy algorithms BOMP and BStOMP clearly deteriorate. In contrast, sparse adaptive algorithms ZA-LMS and ℓ_0-LMS still obtain reliable performance gains, consistent with the conclusion in [18]. The proposed BZA-LMS and BL0-LMS algorithms improve performance slightly, verifying their outstanding noise elimination in robustly solving the CS problem.

Experiment (3): Although the block version of greedy pursuit algorithms BOMP and BStOMP can indeed reconstruct unknown block sparse signals quickly [17], their reconstruction is generally regarded as a type of approximate estimation. In this experiment, we evaluate the computational complexities of the proposed BZA-LMS and BL0-LMS algorithms by comparing them to the BOMP and BStOMP algorithms and to conventional sparse adaptive algorithms ZA-LMS and ℓ_0-LMS. In **Fig. 9**, the running time is measured by the Matlab (R2013a) program running on a Core i5-4120U 64-bit processor, and Windows 10 environment. In various block K-sparsity, i.e. $S \in \{10, 20, \ldots, 80\}$ and $K \in \{1, 2, \ldots, 8\}$, the BOMP and BStOMP algorithms complete block sparse signals reconstruction within 0.02s running time. The ZA-LMS and ℓ_0-LMS algorithms merely cost 5–6 times the time consumption of moderate complexity. The computational complexities of the proposed BZA-LMS and BL0-LMS algorithms remain basically on the same level as that of conventional sparse adaptive algorithms.

5. Conclusions

Based on the fact that evident limitations arise for existing block versions of convex optimization and greedy pursuit algorithms in solving large-scale problem or assuming strong interference. We have extended two sparse adaptive filtering algorithms to block versions, i.e., BZA-LMS and BL0-LMS. In numerical simulation experiments, the effectiveness of the proposed algorithms has been demonstrated for a variety of scenarios.

We concluded that performance has been markedly improved under a variety of block sparsity without sacrificing evident convergence speed. The reliable robustness against strong interference has been investigated and we have shown that our proposed algorithms completely reconstruct block sparse signal while consuming only moderate running time.

Acknowledgements

This work was supported in part by the Japan Society for the Promotion of Science KAKENHI (No. 15K06072).

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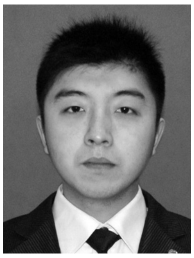

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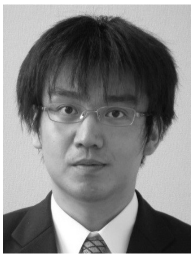

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