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Paper:

Fuzzy Autocorrelation Model with Fuzzy Confidence Intervals and its Evaluation

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Interval models based on fuzzy regression and fuzzy time-series can illustrate the possibilities of a system using the intervals in the model. Thus, the aim is to minimize the vagueness of the model in order to describe the possible states of the system. In the present study, we consider on an interval fuzzy time-series model based on a Box–Jenkins model, a fuzzy autocorrelation model proposed by Yabuuchi, and a fuzzy regressive model proposed by Ozawa. We examine two models by analyzing the Japanese national consumer price index and demonstrate that our approach improves the accuracy of predictions. The utility and predictive accuracy of fuzzy time-series models are validated using two concepts of fuzzy theory and statistics. Finally, we demonstrate the applicability of the fuzzy autocorrelation model with fuzzy confidence intervals.

Keywords: fuzzy time-series model, Box-Jenkins model, autocorrelation, fuzzy random variable

1. Introduction

Time-series models are commonly used in economic analysis, where the objective of the analysis is to understand time-dependent fluctuations in the economic system more precisely based on statistical data. However, economic systems are closely linked to many aggregated factors related to human behavior. Therefore, it is not sufficient to interpret an economic system based only on the results obtained using conventional statistical methods. Instead, when we analyze economic systems that depend on many vague factors, it is desirable to apply the concept of fuzzy theory, which can handle the ambiguity of a structure.

Two types of fuzzy time-series approaches can extend the Box–Jenkins model to deal with fuzzy data and numeric data. The present study focuses on models that deal with fuzzy numbers, such as those proposed by Ozawa et al. [1] and Yabuuchi et al. [2, 3]. Ozawa et al. [1] built a fuzzy autoregressive (FAR) model that includes a differenced series of fuzzified data in its intervals. Yabuuchi et al. [2, 3] defined fuzzy autocorrelation (FAC) coefficients based on fuzzified data and used these values to build a FAC model. Moreover, Yabuuchi proposed an approach for improving the accuracy of predictions using fuzzy random variables [4, 5]. Fuzzy autoregressive integrated moving average (ARIMA) models such as those proposed by Tseng et al. [6, 7] can deal with numeric data. These models express the possibilities of a time-series system based on the fuzzy coefficients of the model in a similar manner to other fuzzy time-series models.

In this study, we consider on a fuzzy time-series model that deals with fuzzy data. We also examine the accuracy of the predictions obtained by a FAR model and a FAC model with and without fuzzy confidence intervals (FCI), as well as demonstrating the utility of these models based on a numerical example. The Japanese national consumer price index is employed in this numerical simulation. We verify the characteristics and usefulness of FAC models.

The remainder of this paper is organized as follows. Section 2 introduces a fuzzy time-series model, which is an extension of the Box–Jenkins model. Section 3 describes the FCI for fuzzy random variables. The FCI is employed instead of time-series data to improve the accuracy of predictions. In Section 4, we use these models to analyze the monthly Japanese national consumer price index from January 1970 to March 2012, and we discuss the results of the analysis.

2. Fuzzy Time-Series Model

The Box–Jenkins time-series model has high predictive precision and it is also easy to handle. In possibility theory, data are considered to be the embodied possibilities of a system. It is natural to consider time-series data as realizations of the possibilities of the time-series system. Therefore, a fuzzy time-series model is an effective means of analyzing time-series systems that include vagueness.

In this section, we introduce two Box–Jenkins-based models; a FAR model and our FAC model. Both models use fuzzy time-series data and they have interval outputs.

2.1. FAR Model

Ozawa et al. [1] proposed a FAR model that expresses the possibilities of fuzzified difference sequences. The fuzzy time-series data $\boldsymbol{Y}_t$ used by the FAR model are fuzzified versions of the original time-series data y_t. The fuzzy data are given as follows:

$$\left.\begin{aligned} \boldsymbol{Y}_t &= [Y_t^L, Y_t^C, Y_t^U], \\ Y_t^U &= y_t \frac{\max(y_{t-1}, y_t, y_{t+1})}{\min(y_{t-1}, y_t, y_{t+1})}, \\ Y_t^C &= y_t, \\ Y_t^L &= y_t \frac{\min(y_{t-1}, y_t, y_{t+1})}{\max(y_{t-1}, y_t, y_{t+1})}. \end{aligned}\right\} \quad \ldots\ldots\ (1)$$

A FAR model is obtained by using the differenced series $\boldsymbol{Z}_t = [Z_t^L, Z_t^C, Z_t^U]$, which removes trends from the fuzzy time-series data $\boldsymbol{Y}_t$ with Eq. (1). The FAR model illustrates the relationship between the fuzzy time-series data $\boldsymbol{Z}_t$ using a real-valued autoregressive parameter ϕ and it represents the vagueness of the system by introducing a triangular fuzzy number $\boldsymbol{u} = [u^L, u^C, u^U]$. The FAR model can be described as follows:

$$\begin{aligned} \tilde{\boldsymbol{Z}}_t &= [\tilde{Z}_t^L, \tilde{Z}_t^C, \tilde{Z}_t^U] \\ &= \phi_1 \boldsymbol{Z}_{t-1} + \phi_2 \boldsymbol{Z}_{t-} + \cdots + \phi_p \boldsymbol{Z}_{t-p} + \boldsymbol{u}. \end{aligned} \quad \ldots\ (2)$$

The autoregressive parameters $\boldsymbol{\phi} = [\phi_1, \phi_2, \ldots, \phi_p]$ are real values and the error term $\boldsymbol{u} = [u^L, u^C, u^U]$ is an asymmetric triangular fuzzy number.

The FAR model (2) results in a linear programming (LP) problem that involves minimizing the ambiguity of the model according to the inclusion relation $\boldsymbol{Z}_t \subseteq \tilde{\boldsymbol{Z}}_t (t = p+1, p+2, \ldots, n)$ as follows:

$$\begin{aligned} \min_{\boldsymbol{\phi}, \boldsymbol{u}} \quad & \sum_{t=p+1}^{n} (\tilde{Z}_t^U - \tilde{Z}_t^L) \\ \text{s.t.} \quad & \boldsymbol{Z}_t \subseteq \tilde{\boldsymbol{Z}}_t \ (t = p+1, p+2, \ldots, n). \end{aligned}$$

2.2. FAC Model

In our FAC model, the time-series data z_t are transformed into a fuzzy number to express the possibilities of the data using a different definition from that given by Ozawa (Eq. (1)). The following fuzzy equation describes the case where only one time point before and after t are considered when building a fuzzy number [8].

$$\begin{aligned} \boldsymbol{Y}_t &= [Y_t^L, Y_t^C, Y_t^U] \\ &= [\min(z_{t-1}, z_t, z_{t+1}), z_t, \max(z_{t-1}, z_t, z_{t+1})] \end{aligned} \quad (3)$$

We use this fuzzy equation to determine the fuzzy time-series data. Furthermore, we employ the calculus of finite differences to filter out the time-series trend data, which allow us to use the following first-order difference equation:

$$\left.\begin{aligned} \boldsymbol{Z}_t &= [Z_t^L, Z_t^C, Z_t^U], \\ Z_t^U &= \max(\boldsymbol{Y}_t - \boldsymbol{Y}_{t-1}), \\ Z_t^C &= Y_t^C - Y_{t-1}^C, \\ Z_t^L &= \min(\boldsymbol{Y}_t - \boldsymbol{Y}_{t-1}). \end{aligned}\right\}$$

FAC models employ a fuzzy operation. In general, the product of negative values increases the vagueness.

2.2.1. FAC

We define the fuzzy autocovariance $\boldsymbol{\Lambda}_k = [\lambda_k^L, \lambda_k^C, \lambda_k^U]$ and the FAC $\boldsymbol{r}_k = [\rho_k^L, \rho_k^C, \rho_k^U]$ using the fuzzy time-series data $\boldsymbol{Z}_t$ and $\boldsymbol{Z}_{t-k}$ as follows:

$$\begin{aligned} \boldsymbol{\Lambda}_k &\equiv Cov[\boldsymbol{Z}_t, \boldsymbol{Z}_{t-k}] = [\lambda_k^L, \lambda_k^C, \lambda_k^U], \\ \boldsymbol{r}_k &= \frac{\boldsymbol{\Lambda}_k}{\boldsymbol{\Lambda}_0} = [\rho_k^L, \rho_k^C, \rho_k^U]. \end{aligned}$$

When we employ a fuzzy operation to obtain a FAC coefficient, the ambiguity may be increased by the fuzzy operation itself. To solve this problem, we adjust the width of a fuzzy number by using an α-cut when determining the difference series. This means that it is impossible to obtain a FAC that reflects the possibilities of the time-series system. Therefore, we maximize the width of the autocorrelation to cover the possibilities of the time-series system. However, the width is determined automatically-because the autocorrelation value should be in the range of $[-1, 1]$. The α-cut level can be obtained by solving the following LP problem:

$$\left.\begin{aligned} \max_{h} \quad & \sum_{i=1}^{p} (\rho_i^U - \rho_i^L) \\ \text{s.t.} \quad & -1 \le \rho_i^L, \rho_i^U \le 1 \\ & \rho_i^L \le \rho_i^C \le \rho_i^U \quad (i = 1, 2, \ldots, p). \end{aligned}\right\}$$

We control the ambiguity of the difference series by employing the α-cut level h, which is obtained by solving the above LP problem. Using the FAC coefficient, which is calculated from the α-cut level h, we redefine the fuzzy Yule–Walker equations as an LP problem, and we calculate the fuzzy partial autocorrelation. The fuzzy Yule–Walker equations are defined below.

2.2.2. Model Definition

We develop the following autoregressive process:

$$\tilde{\boldsymbol{Z}}_t = \boldsymbol{\Phi}_1 \boldsymbol{Z}_{t-1} + \boldsymbol{\Phi}_2 \boldsymbol{Z}_{t-2} + \cdots + \boldsymbol{\Phi}_p \boldsymbol{Z}_{t-p}. \quad \ldots\ (4)$$

Here, $\boldsymbol{\Phi} = [\phi^L, \phi^C, \phi^U]$ denotes a FAR coefficient.

As mentioned above, the next observation value depends on the present observed value; thus, autocorrelation is important for the time-series analysis. Therefore, we build a model that illustrates the ambiguity of the system captured by the FAC. The autocorrelation is also a fuzzy number, so the Yule–Walker equations can be viewed as a fuzzy equation. The fuzzy Yule–Walker equations are written as follows:

$$\boldsymbol{R}_t = \boldsymbol{\Phi}_1 \boldsymbol{r}_{t-1} + \boldsymbol{\Phi}_2 \boldsymbol{r}_{t-2} + \cdots + \boldsymbol{\Phi}_p \boldsymbol{r}_{t-p}. \quad . \quad . \quad . \quad (5)$$

We build the model in terms of FAC, which can describe the ambiguity of the system. However, when the ambiguity of model $\tilde{\boldsymbol{Z}}_t$ is large, the relationship between the model and the system becomes ambiguous. Therefore, the possibilities of the system cannot be described correctly. Hence, to obtain the FAR coefficient for which the ambiguity of a time-series model should be minimized, we have the following LP problem:

$$\left.\begin{array}{ll} \min\limits_{\boldsymbol{\Phi}} & \sum\limits_{i=1}^{p} (\rho_t^U - \rho_t^L) \\ \text{s.t.} & \boldsymbol{r}_t \subseteq \boldsymbol{R}_t, \\ & \rho_t^C = R_t^C \quad (t = 1, 2, \ldots, p). \end{array}\right\} \quad . \quad . \quad . \quad (6)$$

As mentioned above, $\boldsymbol{R}$ is obtained by using a fuzzy operation with the FAC $\boldsymbol{r}$ and FAR coefficient $\boldsymbol{\Phi}$. R^L, R^C, and R^U represent the lower limit, center, and upper limit of $\boldsymbol{R}$, respectively.

FAC models express the possibility that a system change is realized in the data, which is different from the conventional statistical method. Our method can build a model that describes ambiguous portions called possibilities, which are not clearly expressed using conventional statistical techniques.

3. FCI

After obtaining the FAC coefficients using a fuzzy operation, it is easy to obtain a FAC model using LP. However, the ambiguity may increase when we use fuzzy operations. Therefore, we employ a fuzzy random variable to solve the problem of increasing ambiguity. The accuracy of predictions can be improved by using a confidence interval for the fuzzy random data in a FAC model [2].

Various studies have considered fuzzy random variables [9–12], particularly the fuzzy random variables defined by Kwakernaak [13, 14] and Puri et al. [15]. In many cases, fuzzy numbers can be treated as fuzzy random variables. Similarly, it is appropriate to handle fuzzy time-series data as fuzzy random variables. Therefore, our FAC model employs the confidence intervals of fuzzy random variables.

A Box–Jenkins model constructed using an autocorrelation coefficient is likely to overreact to the original series. Our FAC model also overreacts, so we aim to suppress the overreaction of the model. Hence, we use the definition given by Watada [16], which is easy to handle.

3.1. Expected Value and Variance of Fuzzy Random Variables

For the fuzzy variable Y with a possibility distribution μ_Y, the possibility $Pos\{Y \leq r\}$, necessity $Nec\{Y \leq r\}$, and credibility $Cr\{Y \leq r\}$ of an event $\{Y \leq r\}$ are given as follows:

$$Pos\{Y \leq r\} = \sup_{t \leq r} \mu_Y(t),$$
$$Nec\{Y \leq r\} = 1 - \sup_{t > r} \mu_Y(t),$$
$$Cr\{Y \leq r\} = \frac{1}{2}\left(1 + \sup_{t \leq r} \mu_Y(t) - \sup_{t > r} \mu_Y(t)\right).$$

Based on the credibility measure, the expected value of the fuzzy variable is calculated as follows:

$$E[Y] = \int_0^{\infty} Cr\{Y \geq r\} dr - \int_{-\infty}^{0} Cr\{Y \leq r\} dr.$$

Let X be a fuzzy random variable in a probability space (Ω, Σ, Pr), where for each $\omega \in \Omega$, $X(\omega)$ denotes a fuzzy variable. For any fuzzy random variable X on Ω, for each $\omega \in \Omega$, $X(\omega)$, the expected value of the fuzzy variable $X(\omega)$ is denoted by $E[X(\omega)]$. This is known to be a measurable function of ω, i.e., it is a random variable.

Therefore, the expected value of X is defined as follows:

$$E[X] = \int_{\Omega} E[X(\omega)] Pr(\omega).$$

Let X be a fuzzy random variable defined on a probability space (Ω, Σ, Pr) with the expected value e. The variance of X can be defined as follows:

$$V[X] = E[(X - e)^2].$$

3.2. FAC Model with Confidence Intervals

In this study, we employ confidence intervals instead of fuzzy time-series data. These confidence intervals combine the expected value, variance of fuzzy random variables, and the fuzzy inclusion relation at level h to deal with model (4). For instance, to retain more complete information regarding the fuzzy random data, we can employ the fuzzy inclusion relation directly for the product between a fuzzy parameter and a fuzzy value at some probability level. However, this calculation could be difficult because the product of two triangular fuzzy numbers does not retain the same triangular shape as the resulting membership function. Thus, the solution to the problem may be found using heuristics, as proposed by Watada et al. [16].

Before building the FAC model with confidence intervals, we define the confidence interval induced by the expectation e_Z and variance σ_Z^2 of a fuzzy random variable Z. A one-sigma $(1 \times \sigma)$ confidence interval for each fuzzy random variable can be expressed as follows:

$$I[e_Z, \sigma_Z] = [e_Z - \sigma_Z, e_Z + \sigma_Z]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this analysis, we employ confidence intervals for the fuzzy random variable instead of fuzzy times-series data. We refer to these as FCI, and we define a FAC model with FCI as follows:

$$\tilde{\boldsymbol{Z}}_t = [a_1^L, a_1^U] \boldsymbol{Z}_{t-1} + \cdots + [a_p^L, a_p^U] \boldsymbol{Z}_{t-p}, \quad . \quad . \quad . \quad (8)$$

where $[a^L, a^U]$ denotes the coefficient of the FAC model with FCI.

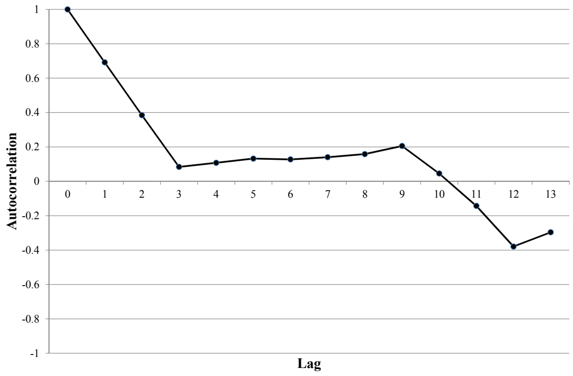

Fig. 1. Correlogram of the Japanese national consumer price index.

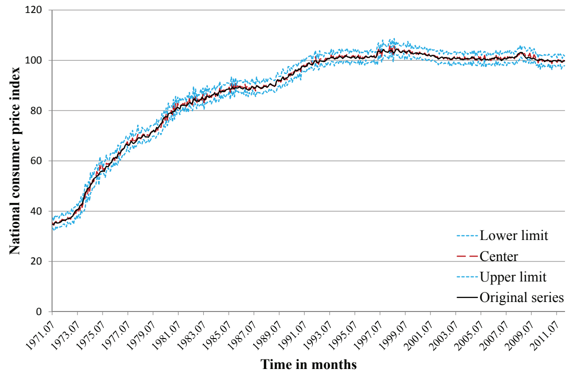

Fig. 2. FAR model and original series.

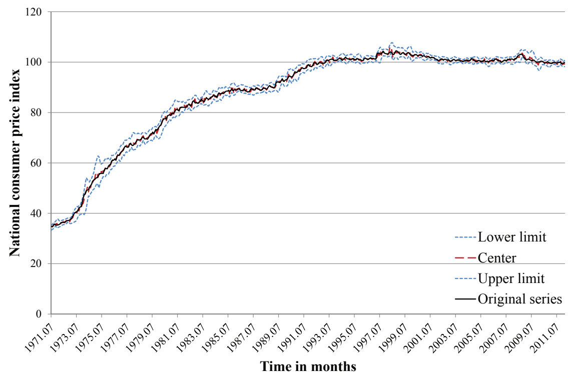

Fig. 3. FAC model and original series.

Our FAC model uses triangular fuzzy numbers, which give a triangular fuzzy autoregressive coefficient, but the autoregressive coefficient is a symmetric triangular fuzzy number when a FCI is used as fuzzy time-series data.

4. Analysis of the Japanese National Consumer Price Index

The 2010-base Japanese national consumer price index was analyzed using a FAR model and FAC models with and without FCI. This consumer price index considers monthly data from January 1970 to March 2012.

In this analysis, we examined three models and identified their utility. The fuzzy autocorrelation model with fuzzy confidence intervals (FCI model) uses a one-sigma confidence interval, so the widths of the data were reduced to 68.3% of that of the original fuzzy time-series data $\boldsymbol{Y}_t$ for the FAR and FAC models.

We define the deviation $\Delta_3\Delta_{12}\boldsymbol{Z}_t - Z^C$ of the differenced series $\Delta_3\Delta_{12}\boldsymbol{Z}_t$, and denote it as $\boldsymbol{X}_t$. Then, we can obtain the fuzzy autocovariance $\boldsymbol{\Lambda}_k$ and the FAC $\boldsymbol{r}_k$ of $\boldsymbol{X}_t$ as follows:

$$\boldsymbol{\Lambda}_k = E[\boldsymbol{X}_t\boldsymbol{X}_{t-k}],\ \boldsymbol{r}_k = \frac{\boldsymbol{\Lambda}_k}{\boldsymbol{\Lambda}_0}.$$

Fig. 1 shows the correlogram of $\boldsymbol{X}_t$. This figure also shows that the one-time previous value and the two-time previous values exhibit a high degree of correlation. Two definitions were used to transform the original series data into fuzzy time-series data. However, the same center was obtained from both definitions. Therefore, both FAR and FAC employed a two-order model. The resulting FAR model is:

$$\begin{aligned}\boldsymbol{X}_t^{FAR} =& 1.390\boldsymbol{X}_{t-1} - 1.117\boldsymbol{X}_{t-2}\\ &+[-1.754, 0.157, 2.067].\end{aligned}$$

Fig. 2 shows the values predicted by the FAR model and the original series.

Next, the FAC model was obtained by solving the fuzzy Yule–Walker equations (6). The resulting FAC model is:

$$\begin{aligned}\boldsymbol{X}_t^{FAC} =& [-0.589, 0.816, 0.816]\boldsymbol{X}_{t-1}\\ &+[-0.219, -0.180, 0.118]\boldsymbol{X}_{t-2}.\end{aligned}$$

Fig. 3 shows the relationship between the values predicted by the FAC model and the original series. Comparing **Figs. 2** and **3**, we can see that the ambiguity of the FAC model is subjectively less than that of the FAR model. However, the center of the FAC model also overreacts to the data. Therefore, it appears that an overreaction to the behavior of the data is a characteristic of models with autocorrelation coefficients.

Adjusting the width of the fuzzy time-series data causes both limits of the fuzzy coefficient of $\boldsymbol{X}_{t-2}$ to become large and positive, which implies that the time-series of the two-time previous values spreads in a positive direction. **Fig. 3** confirms this that the ambiguity of the model decreases as time progresses.

Figure 4 shows the center of the correlogram of the FAC coefficients with FCI. The one-time previous value and the two-time previous values exhibit a high degree of correlation. Similar to the FAC model, the FCI model was obtained by solving the fuzzy Yule–Walker equation. The resulting FCI model is shown in **Fig. 5**, which can be expressed as follows:

$$\begin{aligned}\boldsymbol{X}_t^{FCI} =& [-0.170, 0.956]\boldsymbol{X}_{t-1}\\ &+[-0.141, -0.076]\boldsymbol{X}_{t-2}.\end{aligned}$$

According to **Figs. 3** and **5**, we can confirm that the FCI model has a smaller coefficient width than the FAC model. The FCI model also overreacts, but the degree of overreaction is obviously smaller than that of the FAC model.

The center of the FAC model coincides with that of the

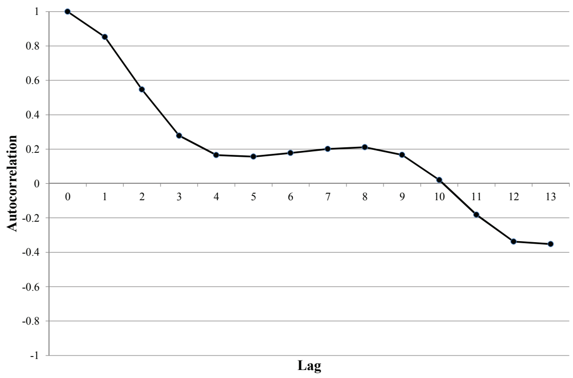

Fig. 4. Correlogram of the FCI.

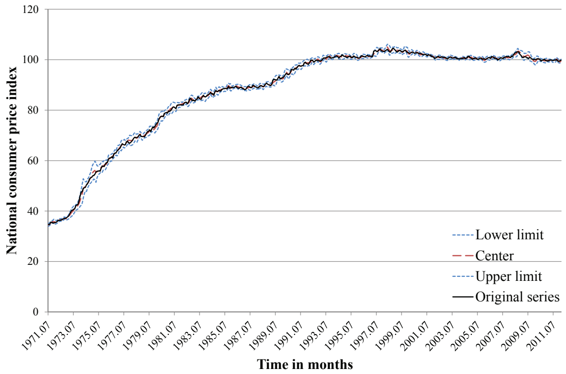

Fig. 5. FCI model and original series.

Table 1. Features of the FAR, FAC, and FCI models.

	FAR model	FAC model	FCI model
Sum of widths of model coefficient	3.821	1.742	1.191
Sum of possibility grades by model and data	444.857	385.875	332.404
Residual sum of squares by centers of model and data	269.197	134.032	86.58
Widths of forecast values	2277.233	1757.700	1023.942

autoregressive model because the FAC coefficient was obtained using a fuzzy operation. This explains the overreaction of the center of the FAC model. Moreover, as mentioned above, ambiguous data may widen the FAC model.

Compared with time-series data, the fluctuations in the center of the confidence intervals are smooth because the center of the FCI is the expectation of fuzzy random variables. Therefore, the overreaction of the FAC model with confidence intervals is absorbed.

The features of the FAR, FAC, and FCI models are listed in **Table 1**. Although these are fuzzy models, the residual sum of squares was used to evaluate the estimates.

First, it can be seen that the FAC model is more accurate than the FAR model. The residual sum of squares given by the centers of the FAR model is twice that of the FAC and FCI models. The sum of widths for the FAR model is also larger than those for the other models. In general, the possibility grade will be large when the width is large. Hence, the sum of the possibility grades derived from the FAR model is greater than those of the other models.

Second, the FCI model is more accurate than the FAC model, because the residual sum of squares given by the centers of the FCI model is smaller than that of the FAC model. The width of the values predicted by the FAC model is approximately 1758, whereas that for the FCI model is approximately 1024 (see **Table 1**), which implies that the vagueness of the FCI model is less than that of the other models. The sum of possibility grades derived from the FCI model and the data is approximately 0.9 times that given by the FAC model. In addition, the residual sum of squares given by the centers of the FCI model is 0.7 times that of the FAC model. Therefore, the FCI model is more accurate in this time-series system.

These models were built using data from February 1970 to February 2012. To validate the model, we compared the predicted values and the real data for the 12 months from March 2011 to February 2012 and for the next 12 months from March 2012 to February 2013. Thus, the predicted values from March 2011 to February 2012 were used as the real data, and the next 12 months were used the predicted values. **Figs. 6–8** show the results obtained from the model validation process. The width of the FAR model changed periodically and both of the limits switched in August 2012. Thus, as shown in **Fig. 6**, the large value set the upper limit and the small value set the lower limit. The center of the FAC model and the original series exhibited almost the same behavior. However, because FCI use an expected value for a fuzzy random variable, the values predicted by the FCI model reacted slowly to the behavior of the original series. The width of the model reflects its vagueness, so it is obvious that the FCI model is narrower than the FAC model.

The widths of the predicted values are shown in **Table 2**. The FAC model is approximately 1.4 times wider than the FCI model. Again, the FCI is smooth because it uses an expected value. Therefore, it appears that the prediction accuracy of the FCI model is better than that of the FAC model.

The verification of the FAR model was not particularly impressive, so we examined the models using statistical methods. We considerd AR models, so methods such as the augmented Dickey–Fuller test and the Phillips–Perron unit root test could be used. However, because these are

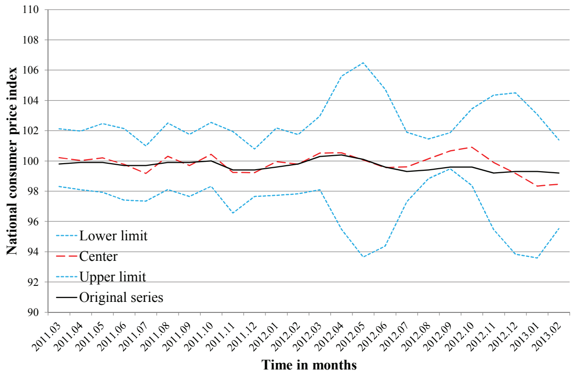

Fig. 6. Validation of the FAR model.

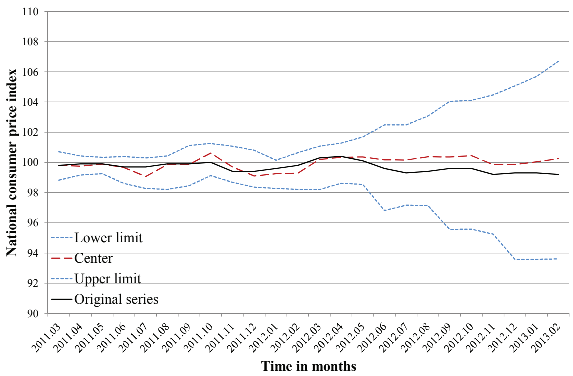

Fig. 7. Validation of the FAC model.

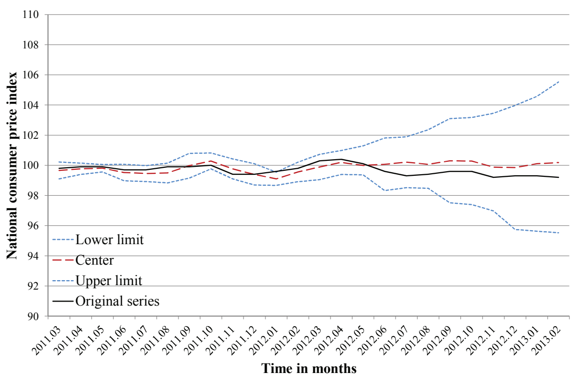

Fig. 8. Validation of the FCI model.

technical methods, we tested the models using a t-test and a correlogram of the residual errors. **Table 3** presents the results of the t-test, which were obtained using the residual errors. This table indicates that the FAC model and the FCI model can be considered the same as the original series. However, the center of the FAR model does not correspond with the original series.

Next, the results of the t-test were confirmed using the correlograms. Large autocorrelation coefficients are visible in **Fig. 9**, but they are within the 95% confidence intervals. However, it seems that the FAR model exhibits a different trend to the other models. The data and model settings may be unstable, but the FAR model does not represent the time-series system with these data.

Table 2. Widths of predicted values.

	FAR model	FAC model	FCI model
2012/03	7.114	3.030	1.672
2012/04	14.828	2.784	1.587
2012/05	18.789	3.308	1.914
2012/06	15.171	5.960	3.483
2012/07	6.715	5.564	3.378
2012/08	3.838	6.240	3.879
2012/09	3.500	8.897	5.580
2012/10	7.413	8.959	5.783
2012/11	13.001	9.720	6.472
2012/12	15.609	12.088	8.241
2013/01	13.887	12.765	8.948
2013/02	8.575	13.844	10.005

The three models were verified statistically using the centroid of the fuzzy estimates obtained by each model, where we traced the data distribution at its centroid. Therefore, the centroid of the predicted values can be considered to represent the predictive accuracy of the model. However, the FCI model has a symmetric triangular fuzzy numbers as fuzzy coefficients, so its characteristic values are the same when using a centroid or a center value. The characteristic values obtained using the centroid of each estimated value are shown in **Table 4**. The correlation coefficient of each model is approximately 1. Using the centroid of the predicted values and data, the FCI model has the smallest sum of squared deviations. In fact, the FCI model has the smallest ambiguity among the three models considered and its predictive accuracy is high. Moreover, the FAR model has twice the width of the FAC model.

According to the results descrived above, we can confirm that highly accurate time-series predictions can be obtained using FCI. In addition, the use of a FCI decreases the model vagueness compared with using fuzzy time-series data. In the case of the Japanese national consumer price index, the vagueness of the FCI model was only 0.71 times that of the vagueness of the FAC model. In this study, we used the centroid for the evaluation interval of the forecast values, and the results were subjectively acceptable.

Based on the results in **Tables 1** and **2**, the FCI model had the best predictive accuracy. For the centroid-based evaluation, the FCI model has an advantage over the other two models because it uses the fuzzy coefficients of symmetric triangular fuzzy numbers. Therefore, when the fuzzy coefficients are of different types, it is better to use a possibility grade derived from models, samples, and the model widths.

5. Conclusions

In this analysis, the Japanese consumer price index was described by the one-time previous value and the two-time previous values using FAR and FAC models, which were extended versions of the Box–Jenkins model. The

Table 3. Statistical information from a model test.

	FAR model	FAC model	FCI model
t-value	-4.8074	-0.0148	-0.086
p-value	0.0000	0.9882	0.9315
95% confidence interval	$[-0.2227, -0.0935]$	$[-0.0470, 0.0463]$	$[-0.0391, 0.0359]$
Mean of the residual error	-0.1581	-0.0004	-0.0016

Table 4. Characteristic values obtained using the centroid of each estimated value.

	FAR model	FAC model	FCI model
Correlation coefficient obtained using centroid and data	0.9993	0.9997	0.9998
Sum of squared deviation obtained using centroid and data	268.8509	128.3598	86.5810

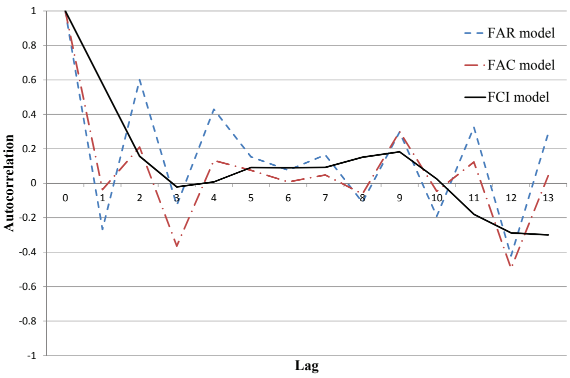

Fig. 9. Autocorrelation of the residual error of the models.

FAC model was also applied with FCI. We examined these models using statistical methods.

We confirmed that highly accurate time-series predictions can be obtained using FCI. In addition, the vagueness of the FCI model was less than that of the models based on fuzzy time-series data. In fact, the vagueness of the FCI model was only 0.71 times that of the FAC model.

In addition, the centroids of the estimated values derived from the models and data were evaluated statistically. The centers of the fuzzy coefficients of the one-time previous value and two-time previous values were $(0.816, -0.180)$ for the FAC model and $(0.393, -0.109)$ for the FCI model. This indicates that the consumer price index is positively influenced by the previous month and negatively influenced by the month before that. Thus, the effect from two months ago removes the effect of the last month. However, the fuzzy coefficient of the FAR model was $(-1.754, 0.157)$, which has the opposite sign to the FAC and FCI models. Although the results predicted by the FAR model appear to be highly accurate, the applicability of the FAR model obtained was not validated by the t-test.

To consider the probability of the occurrence of fuzzy time-series data, the center of the FCI contains smoothed values, so the FCI helps to improve the model accuracy when the model overreacts, .

However, our comparison of models using symmetric and asymmetric triangular fuzzy numbers as fuzzy coefficients showed that the model using asymmetric triangular fuzzy number was most the advantageous. Therefore, the use of centroids to compare various models should be discouraged. An index that considers fuzzy concepts, such as the possibility grades of a model and the samples and widths of the model, is more suitable for model comparisons.

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