

Paper:

# Optimal Outpatient Appointment System with Uncertain Parameters Using Adaptive-Penalty Genetic Algorithm

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**The optimal number of doctors and appointment interval for an outpatient appointment system in a class of individual block/fixed interval are determined using an adaptive-penalty Genetic Algorithm. The length of service time for doctor consultation, the time required for the laboratory tests, and the time deviating from the appointment time are modelled by random variables. No-show patients are also included in the system. Using the adaptive penalty scheme, optimization constraints are automatically and numerically handled. The solution methodology is readily applicable to other appointment systems. The study has a significant implication from the viewpoint of economic and risk management of health care service.**

**Keywords:** outpatient appointment system, adaptive-penalty GA, uncertainty

## 1. Introduction

Healthcare plays a fundamental and essential role in daily life. Demanding on outpatient service is persistently high. It is widely recognized that the utilization of an appointment system is a systematic and cost-effective way for outpatient service. A typical appointment system that has been globally used is the class of individual block/fixed interval [1]. Outpatient departments are often provided with a number of doctors. Liu and Liu [2] considered two to five doctors in their simulation study of block appointment systems. Yeon et al. [3] investigated the scheduling problem with multi-doctor sharing resources.

Major problems in providing satisfactory healthcare service are the shortage of doctors and the lack of appropriate pre-specified appointment interval for consulting outpatients. The doctor shortage is a direct result from the economic constraints in training new doctors, which takes time and is costly. Therefore, the optimal assign-

ment of suitable doctors to each hospital is required thorough consideration. The appropriate appointment interval is still an interest issue because it can drastically affect the service quality [4]. Consequently, the optimal determination of number of doctors and appointment interval for an appointment system is a crucial aspect when designing or revising an appointment system.

The research on appointment systems has received intensive attention since the pioneering work from Bailey [5]. A comprehensive review of the literature on appointment systems can be found in [1]. The appointment system which is considered here is a typical system used in the hospitals in Thailand and belongs to the class of individual block/fixed interval [1]. Parallel number of doctors is assigned to the system. However, only the patients with the appointments will be exclusively considered and the disturbance against the appointment times from such randomly walk-in patients is thus eliminated. The patient punctuality is defined in terms of the difference between the time of appointment and the time of patient arrival. The arrival times of the appointed patients can be random around the appointed times. Regarding the presence status of an appointed patient, it is possible that some patients will not actually show up for the appointments, i.e. no-show patients. As no-show cases are unavoidable, it is also incorporated into the model of the appointment system. In addition, some patients require second consultation to the same doctors after their laboratory tests. The fact of multiple doctors, no-show patients, and second consultation is simultaneously considered with the objective to simulate real service situations. To reflect the uncertainty, the length of the service time for a doctor consultation, the time required for the laboratory tests, and the time deviating from the appointment time are modelled by random variables. The consideration of uncertainty significantly improves the risk management of health care service.

Various techniques were employed in the optimal design of appointment systems. Fries and Marathe [6] used dynamic programming to determine the optimal block



sizes for the next period given that the number of patients remaining to be assigned is known. Liao et al. [7] applied dynamic programming to determine the optimal block sizes when service times are Erlang. Liu and Liu [8] developed a dynamic programming formulation to find the optimal block sizes in their study on a queuing system with multiple doctors with random arrival times. Pegenden and Rosenshine [9] applied a Markov-chain based procedure to compute the optimal appointment intervals. Robinson and Chen [10] formulated the problem of finding the optimal appointment times as a stochastic linear program and solved it using Monte-Carlo integration. Denton and Gupta [11] presented a two-stage stochastic linear programming model to determine the optimal appointment intervals. Vanden Bosch et al. [12] proposed a fathoming approach to solve the same problem as that of Liao et al. [7]. Kaandorp and Koole [13] introduced a local search procedure to determine the optimal schedule with a weighted average of expected waiting times of patients, idle time of the doctor and tardiness as objective. Güler [14] solved the schedule assignments of the residents and the senior academic staff to outpatient clinics using a hierarchical goal programming. The optimization is integrated into the scheduling of chemotherapy outpatient appointments [15]. However, the treatment time is assumed to be deterministic. In addition, the deviation from the appointment time is not considered, i.e. assuming punctuality for patient arrival. The problem aspects are thus not reflecting real situation. A dynamic appointment scheduling is considered in [16]. The so-called column generation is proposed to solve the problem. The method is, however, limited to the considered problem.

This paper investigates the issue of resource management. There are two novel contributions from the present paper. Firstly, the paper considers the determination of the optimal number of doctors and appointment interval for an outpatient appointment system. The first contribution has not been considered by the literature. Yet, this contribution aspect is especially crucial in practice when the number of doctors needs to be known for the operation of new hospitals or for the financial planning of existing hospitals. Secondly, it is shown in this paper that the complicate nature of the present problem in terms of patient punctuality, patient no-show, two-time examination, laboratory test, and uncertainty can be effectively tackled by a straightforward but powerful optimization tool.

The determination is formulated within the framework of a constrained optimization problem in which the number of doctors and the appointment interval are two design variables. According to the reviewed techniques above, those techniques were applied to the scheduling-type problem and their applicability is limited to those problems. Genetic Algorithm (GA) is proposed herein as an optimization tool.

GA is a class of global search techniques that are inspired by evolutionary theory in biological sciences [17]. A GA was proposed for solving the surgery scheduling problem by Wang et al. [18]. A GA for solving a multi-objective problem of scheduling of radiotherapy treat-

ments for categorized cancer patients was described by Petrovic et al. [19]. A combination of simulation method with GA was used for adjusting the schedule of the nurses in a hospital emergency department by Yeh and Lin [20].

GA is, however, utilized herein for another different aspect, i.e. for determining optimal number of doctors and appointment interval. Since the considered problem involves a number of complicate optimization constraints, an adaptive-penalty GA [21] that can handle multiple and complicate constraints are employed. Such an adaptive-penalty GA has proved its high capability in handling complicate constraints in an effective and efficient manner [22].

The structure of the paper is below. Section 2 presents the modelling of appointment system and definitions of terms relevant to system characteristics and performances. The description of GA and its application to the optimal design towards desirable performances of appointment systems are described in Section 3. Section 4 conducts a numerical example to elucidate the methodology. The conclusion is finally drawn in Section 5.

## 2. Problem Formulation

### 2.1. Definition of Appointment System

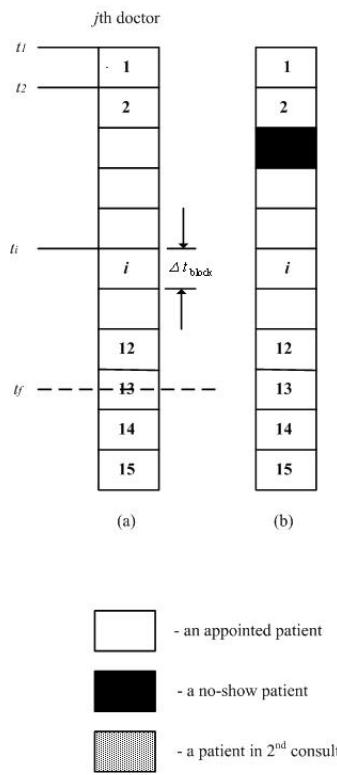
The following definitions, characteristics, and rules are applied to the appointment system of present interest.

1) The appointment system is in the class of parallel individual block/fixed interval.

2) There are  $n_D$  parallel doctors. It is assumed all doctors have same skills and knowledge. For a given number of patients  $N_{patient}$ , the patients will be distributed to each doctor as uniformly as possible. For example, when  $n_D$  is 3 and  $N_{patient}$  is 18, the number of patients assigned to each doctor is equal to 6. However, when  $n_D$  is 3 and  $N_{patient}$  is 20, two doctors take care of 7 patients and one doctor is responsible for 6 patients. The number of patients assigned to the  $j_{th}$  doctor is denoted as  $NO_j$ .

3) It is possible that an appointed patient may be absent. The absence probability of each patient is equally designated to  $p_{abs}$ .

4) It is possible that an appointed patient can have laboratory tests. Each patient has an equal probability of having laboratory tests of  $p_{lab}$ . After the tests, that patient needs another consultation. The second consultation is considered as another additional appointment case under the same doctor. Accordingly, a patient who has laboratory tests creates two consultation cases. If the patient that needs the second consultation arrives at the doctor room before the appointed patient in the schedule, the second-time patient is given a priority to see the doctor. Otherwise, the second-time patient can see the doctor after the appointed patient in the schedule has finished the consultation. In other words, the First-Come-First-Serve (FCFS) principle is used when there is the interruption in the original schedule from second-consultation patients. All present patients have at least one consultation, i.e.



**Fig. 1.** A part of an appointment system for a doctor.

their first consultation cases.

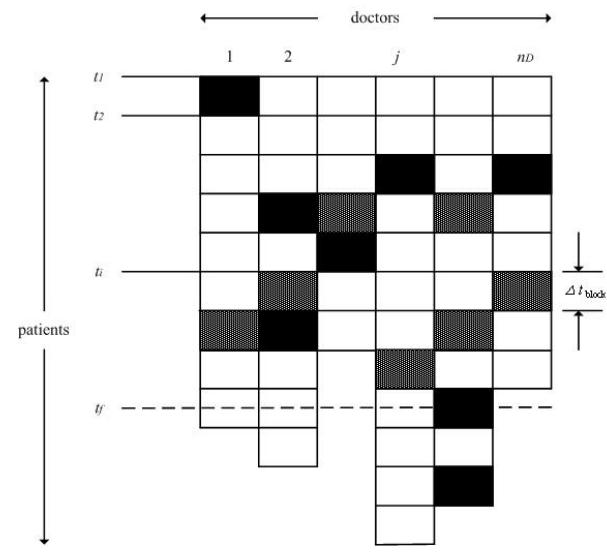
5) Each doctor may have different number of appointment blocks. Each block consists of only one consultation case.  $NP_j$  is the total number of consultation cases, including both first and second consultations, under the  $j_{th}$  doctor. This number counts each second consultation as a case of treatment. Consequently,  $NP_j \geq NO_j$ .

6) The appointment interval is denoted by  $\Delta t_{block}$ . The appointment time at the beginning of the  $i_{th}$  appointment block is designated to  $t_i$ . Without loss of generality,  $t_1$  is set equal to zero.

7) The office hour is ended at  $t_f$ .

The graphical representation of an appointment system is shown in **Figs. 1** and **2**. **Fig. 1** shows an appointment system belonging to a doctor. The system shown in **Fig. 1(a)** does not include the no-show and second-consultation patients. **Fig. 1(b)** depicts an appointment system with a no-show patient. **Fig. 1(c)** includes both no-show and second-consultation patients. It is noted that the original ranks of appointed patients have been changed due to the interference from the patients requiring second consultations. The ranks of the patients subjected to the change are expressed in a format of two numbers. For example, 13(12) means that this patient is originally at the 12<sup>th</sup> rank in the queue but is then shifted to the 13<sup>th</sup> rank because of the interference. **Fig. 2** represents an appointment system with multiple doctors, no-show patients, and patients requiring second consultations.

The variables and terms that are related to the service



**Fig. 2.** An appointment system with multiple doctors, no-show patients, and patients requiring second consultations.

and performance indices are below.

$A_{ij}$ : arrival time of the  $i_{th}$ -block of consultation case (either first or second) under the  $j_{th}$  doctor in the appointment system,

$L_{ij}$ : length of service time for the  $i_{th}$ -block of consultation case (either first or second) under the  $j_{th}$  doctor in the appointment system,

$B_{ij}$ : starting service time of the  $i_{th}$ -block of consultation case (either first or second) under the  $j_{th}$  doctor in the appointment system,

$E_{ij}$ : ending service time of the  $i_{th}$ -block of consultation case (either first or second) under the  $j_{th}$  doctor in the appointment system.

The arrival time of each consultation case is classified into two types. For the first consultation case, the arrival time is related to the appointment time as follows:

$$A_{ij} = t_i + \Delta_{ij}, \dots \quad (1)$$

where  $\Delta_{ij}$  is the time deviating from the appointment time  $t_i$ . The deviation time can be random and thus treated as a random variable. The punctuality of an appointed patient is interpreted from the condition

$$\Delta_{ij} \begin{cases} < 0 & \text{early arrival} \\ = 0 & \text{punctual arrival} \\ > 0 & \text{late arrival} \end{cases} \dots \quad (2)$$

When considering that the earliness or waiting prior to appointment time is not a consequence of the appointment

system as in [1], then  $\Delta_{ij}$  is defined as

$$\Delta_{ij} \begin{cases} = 0 & \text{early and punctual arrival} \\ > 0 & \text{late arrival} \end{cases} \dots \quad (3)$$

For the second consultation case, the arrival time is given by

$$A_{ij} = EF_{ij} + TL_{ij} \dots \dots \dots \quad (4)$$

where  $EF_{ij}$  is the ending service time after the first consultation and  $TL_{ij}$  is the time required for the laboratory tests of that patient, respectively. The ending service time after the first consultation can be computed from Eq. (6).

The starting service time  $B_{ij}$  is obtained from

$$B_{ij} = \max(A_{ij}, E_{(i-1)j}); i = 2, \dots, NP_j \dots \quad (5a)$$

$$B_{ij} = \max(A_{1j}, t_1), \dots \dots \dots \quad (5b)$$

which reflects the fact that the first patient to each doctor can have the health care service only after the starting office hour.

The ending service time of each consultation case, i.e. first or second consultation, is defined as

$$E_{ij} = B_{ij} + L_{ij} \dots \dots \dots \quad (6)$$

$L_{ij}$  is equal to zero if the  $i_{th}$ -block patient under the  $j_{th}$  doctor is absent or no-show. In addition, when the patient under consideration requires the second consultation and  $E_{ij}$  corresponds to the ending service time after the first consultation, then  $E_{ij}$  is further used as  $EF_{ij}$  for the computation of the arrival time for the corresponding second consultation case. That is

$$EF_{ij} = E_{ij} \dots \dots \dots \quad (7)$$

for its used in Eq. (4). It is noted that the length of service time  $L_{ij}$  is separated into two cases in all mathematical expressions. In the first consultation case, the length of service time for the first consultation  $L1_{ij}$  must be used for  $L_{ij}$ , i.e. setting

$$L_{ij} = L1_{ij} \dots \dots \dots \quad (8a)$$

$$L_{ij} = L2_{ij} \dots \dots \dots \quad (8b)$$

The second consultation case fixes the length of service time for the second consultation  $L2_{ij}$  for  $L_{ij}$ .

## 2.2. Indices of System Performance

In this section, the relevant performance indices will be defined. First, the waiting time  $W_{ij}$  of the  $i_{th}$ -block of consultation case (either first or second) under the  $j_{th}$  doctor is

$$W_{ij} = \max(0, B_{ij} - A_{ij}) \dots \dots \dots \quad (9)$$

The total waiting time corresponding to the service from the  $j_{th}$  doctor  $W_j$  is

$$W_j = \sum_{i=1}^{NP_j} W_{ij} \dots \dots \dots \quad (10)$$

The total waiting time in the appointment system  $W_T$  is thus

$$W_T = \sum_{j=1}^{nD} W_j \dots \dots \dots \dots \dots \quad (11)$$

The average waiting time of a patient  $W_A$  is

$$W_A = \frac{1}{N_p n_D} W_T, \dots \dots \dots \dots \dots \quad (12)$$

where

$$N_p = \sum_{j=1}^{nD} NP_j \dots \dots \dots \dots \dots \quad (13)$$

The overtime of the  $j_{th}$  doctor  $OT_j$  is obtained from

$$OT_j = \max(0, E_{NP,j} - t_f), \dots \dots \dots \quad (14)$$

where  $E_{NP,j}$  is the ending service time of the last consultation case under the  $j_{th}$  doctor. The definition of the  $j_{th}$ -doctor overtime implies that there is no overtime if the doctor finishes the work before the office hour.

The total overtime in the appointment system  $OT_T$  is

$$OT_T = \sum_{j=1}^{nD} OT_j \dots \dots \dots \dots \dots \quad (15)$$

The average overtime for a doctor  $OT_A$  is

$$OT_A = \frac{1}{n_D} OT_T \dots \dots \dots \dots \dots \quad (16)$$

The  $j_{th}$  doctor idle time incurred just before the arrival of the  $i_{th}$ -block of consultation case (either first or second) is

$$IT_{ij} = \max(0, A_{ij} - E_{(i-1)j}); i = 2, \dots, NP_j \quad (17a)$$

$$IT_{1j} = \max(0, A_{1j} - t_1) \dots \dots \dots \quad (17b)$$

The total idle time of the  $j_{th}$  doctor  $IT_j$  is

$$IT_j = \begin{cases} \sum_{i=1}^{NP_j} IT_{ij} & ; OT_j \geq 0 \\ \sum_{i=1}^{NP_j} IT_{ij} + |OT_j| & ; OT_j < 0 \end{cases} \dots \dots \dots \quad (18)$$

The inclusion of the overtime term into the computation of the total idle time suggests that the free time of the doctor before the end of the office hour be considered as an idle time as well.

The total idle time in the appointment system  $IT_T$  is

$$IT_T = \sum_{j=1}^{nD} IT_j \dots \dots \dots \dots \dots \quad (19)$$

The average idle time for a doctor  $IT_A$  is

$$IT_A = \frac{1}{n_D} IT_T \dots \dots \dots \dots \dots \quad (20)$$

As mentioned in the previous sections, a number of appointed patients may not appear at the times of appointment and some appointed patients need to have laboratory tests and thus their second consultation. In addition, the

patient punctuality, i.e. the deviation time from the appointment time, can be random. The framework of probability theory will be utilized in this study to model and measure the uncertainty. The inclusion of these random events makes the defined performance indices become uncertainty too. The measurements of the uncertain performance indices will be then carried out using probabilistic measures. Accordingly, the performance of the appointment system is measured through the expected total cost of appointment system  $E[C_T]$  as defined by

$$E[C_T] = c_w E[W - T] + c_{OT} E[OT_T] + c_{IT} E[IT_T], \quad (21)$$

where  $c_w$ ,  $c_{OT}$ , and  $c_{IT}$  are the cost per time units associated to  $W_T$ ,  $OT_T$ , and  $IT_T$ , respectively. The symbol  $E[V]$  denotes the expectation of the random variable  $V$ .

### 2.3. Optimization of Appointment System

The optimization problem of the appointment system is formulated as follows:

$$\begin{aligned} \text{Minimize}_{(n_D, \Delta t_{block})} &= E[C_T(n_D, \Delta t_{block})] |_{N_{\text{patient}}} \\ &= c_w E[W_T(n_D, \Delta t_{block})] |_{N_{\text{patient}}} \\ &\quad + c_{OT} E[OT_T(n_D, \Delta t_{block})] |_{N_{\text{patient}}} \\ &\quad + c_{IT} E[IT_T(n_D, \Delta t_{block})] |_{N_{\text{patient}}} \\ &\quad \dots \end{aligned} \quad (22)$$

Subject to

$$g_1(n_D, \Delta t_{block}) = E[W_A(n_D, \Delta t_{block})] |_{N_{\text{patient}}} - \delta_W \leq 0 \quad \dots \quad (23)$$

$$g_2(n_D, \Delta t_{block}) = E[OT_A(n_D, \Delta t_{block})] |_{N_{\text{patient}}} - \delta_{OT} \leq 0 \quad \dots \quad (24)$$

$$g_3(n_D, \Delta t_{block}) = E[IT_A(n_D, \Delta t_{block})] |_{N_{\text{patient}}} - \delta_{IT} \leq 0, \quad \dots \quad (25)$$

where  $O(n_D, \Delta t_{block})$  is the objective function.  $g_1(n_D, \Delta t_{block})$ ,  $g_2(n_D, \Delta t_{block})$ , and  $g_3(n_D, \Delta t_{block})$  are the inequality constraints.  $\delta_W$ ,  $\delta_{OT}$ , and  $\delta_{IT}$  are the thresholds of the average waiting time of a patient, the average overtime for a doctor, and the average idle time for a doctor, respectively.

The design variables that minimize the objective function (22) and at the same time satisfy the constraints (23) to (25) will be referred to as the optimal number of doctors  $n_D^*$  and the optimal appointment interval  $\Delta t_{block}^*$ .

## 3. Genetic Algorithm (GA)

### 3.1. General on GA

GA is a class of global and stochastic search techniques that are founded in the mechanism of natural selection. The GA procedure starts with an initial set of randomly selected trial solutions, namely population. Each individual in the population is encrypted and referred to as a chromosome which represents a possible solution

to the optimization problem. The chromosomes evolve through successive iterations, called generations. In each generation, the fitness of each chromosome is evaluated. The fitness of each chromosome reflects the potential to be the optimal solution. Each chromosome is reproduced according to its fitness value. Fitter chromosomes have higher probabilities to be selected for reproduction whereas weaker chromosomes tend to die off. The chromosome selection and reproduction are carried out in a reproduction process. The chromosomes resulting from the reproduction process form a mating pool and are collectively referred to as offspring. The offspring are later undergone genetic operations. The exploration of search space is carried out through the genetic operations where genetic operators are applied to existing chromosomes and transform them into new chromosomes. The genetic operators-derived chromosomes represent new trial solutions in the search space. The resulting chromosomes then form the new generation of population. It should be noted that GA works in two spaces alternatively. The selection process is performed in the space of original variables while the genetic operations are done in the space of coded variables. Both spaces are referred to as the solution and coding space, respectively [14]. The GA search is terminated when a prescribed number of generations have elapsed.

### 3.2. Chromosome Representation

GA encrypts each trial solution into a sequence of numbers or strings and denotes the sequences as a chromosome. A simple binary coding [16] is widely known and will be employed for coding the optimal number of doctors  $n_D$  and the appointment interval  $\Delta t_{block}$ .

### 3.3. Reproduction Process

Reproduction in GA is a process in which individual chromosomes are copied according to their fitness values. This operation imitates the survival of the fittest or the natural selection following Darwin's principle [23]. Fitness in an optimization by GA is defined by a fitness function. Based on the optimization problem as described by the objective function (22) and the set of constraints (23) to (25), the fitness function  $F(x)$  of a chromosome representing a vector  $x$  of design variables in the solution space is defined as

$$F(x) = \begin{cases} O(x) & ; x \text{ is feasible} \\ O(x) - \sum_{j=1}^{NC} k_j v_j(x) & ; x \text{ is infeasible}, \end{cases} \quad (26)$$

where  $v_j(x)$  is the violation magnitude of the  $j_{th}$  constraint,  $k_j$  is the penalty parameter for the  $j_{th}$  constraint defined at each generation and  $NC$  is the number of constraints.

An adaptive penalty scheme which is introduced by Barbosa and Lemonge [24] and improved by Obadage and Hampornchai [21] is modified in handling the constraints. The improved adaptive penalty scheme shows its

excellent capability in handling a very large number of constraints [22]. This adaptive scheme is given by

$$k_j = |\max(O^{inf}(x))| \frac{\langle v_j(x) \rangle}{\sum_{l=1}^{NC} [\langle v_l(x) \rangle]^2} \quad \dots \quad (27)$$

where  $\max(O^{inf}(x))$  is the maximum of the objective function values in the current population in the infeasible region,  $v_j(x)$  is the violation magnitude of the  $j_{th}$  constraint.  $\langle v_j(x) \rangle$  is the average of  $v_j(x)$  over the current population. The violation magnitude is defined as

$$v_1(x) = \begin{cases} |g_1(x)| & ; g_1(x) > 0 \\ 0 & ; \text{otherwise} \end{cases} \quad \dots \quad (28)$$

The reproduction operator may be implemented in a number of ways. The easiest and well-known approach is the roulette-wheel selection [25, 26]. According to the roulette-wheel scheme, the  $j_{th}$  chromosome will be reproduced with the probability of

$$P_j = \frac{F_j}{\sum_{l=1}^{N_{Pop}} F_l}, \quad \dots \quad (29)$$

where  $N_{Pop}$  is the population or sample size. The fitness value  $F_j$  is obtained from Eq. (26).

### 3.4. Genetic Operators

GA utilizes two genetic operators, namely crossover and mutation. Both operators have different mechanisms but aim at the same task which is exploring the solution space in order to attain optimal values. The crossover operator involves the swapping of genetic material (bit-values) between the two parent strings. This operator randomly chooses a locus (a bit position along the two chromosomes) and exchanges the sub-sequences before and after that locus between two chromosomes to create two offspring.

A simple binary crossover with two cut points, namely two-point crossover performs the search by randomly selected two cut points along the chromosomes. The strings in between the cut points of both chromosomes are then exchanged each other. The crossover rate  $P_c$  is defined as the ratio of the number of chromosome that will be crossovered to the population size.

Mutation randomly flips or alters one or more bit values at randomly selected genes in a chromosome. The values of the selected genes are then converted either from 0 to 1 or from 1 to 0. The mutation rate  $P_m$  is defined as the ratio of the number of genes that will be mutated to the population size. The mutation operation also assists the exploration for potential solutions which may be overlooked by the crossover operation.

**Table 1.** Definition of random variables in the numerical example.

Random Variable	Distribution (minutes)
$L1_{ij}$	Uniform (10,20)
$L2_{ij}$	Uniform (7,12)
$TL_{ij}$	Triangular (10,20,30)
$D_{ij}$	Uniform (0,10)

Length of service time for the 1st-consultation ( $L1_{ij}$ ), length of service time for the 2nd-consultation ( $L2_{ij}$ ), time required for the laboratory tests ( $TL_{ij}$ ), time deviating from the appointment time ( $D_{ij}$ ).

**Table 2.** Threshold values in case of deterministic number of patients.

Parameter	Magnitude (minutes)
$d_W$	5
$d_{OT}$	30
$d_{IT}$	30

## 4. Numerical Example

The appointment system described in Section 2 is considered in this section.  $c_W$ ,  $c_{OT}$ , and  $c_{IT}$  are set equal to 100, 600, and 300, respectively. The absence or no-show probability of each patient  $p_{abs}$  is equal to 0.20. The probability that an appointed patient will have laboratory tests  $p_{lab}$  is equal to 0.40. The ending time of the office hour  $t_f$  is equal to 180. All random variables are assumed statistically independent and their descriptions are given in **Table 1**.

The values of the thresholds are given in **Table 2**.

The distributions in **Table 1** and the threshold values in **Table 2** can be obtained from data collection and statistical analysis. The data collection can be obtained from existing hospitals in the health care system.

GA search employs the population size of 100. The number of generations used in the search is 100. The employed population size and the generation number yield stationary result in the average of the feasible solutions. Monte Carlo Simulation (MCS) is run for each individual chromosome to obtain its corresponding value of the fitness function. A two-point crossover is utilized.  $P_c$  consists of 0.6, 0.7, 0.8, 0.9 where as  $P_m$  includes 0.005, 0.004, 0.003, 0.002, 0.001. **Table 3** summarizes  $n_D^*$  and  $\Delta t_{block}^*$  for different values of  $N_{patient}$  including the corresponding GA parameters.

It is obvious that the constraints in this numerical example are complicated and can be obtained in terms of numerical values only. Nevertheless, the adaptive scheme can handle the constraints in an automatic manner and thus avoid the need of a priori knowledge about the characteristics of the constraints. The numerical results show that the optimal number of doctor  $n_D^*$  play more important role than the optimal appointment interval  $\Delta t_{block}^*$  since  $\Delta t_{block}^*$  is considerably less sensitive, compared with  $n_D^*$ , to the change in the expected number of patients  $N_{patient}$ .

**Table 3.** Optimal appointment systems for various number of patients.

$N_{patient}$	$P_c$	$P_m$	$n_D^*$	$\Delta t_{block}^*$
50	0.7	0.001	5	15
100	0.7	0.002	10	14
200	0.8	0.002	20	12
300	0.8	0.002	30	14
400	0.8	0.002	40	12

**Table 4.** Optimization results using fixed penalty parameters for  $N_{patient} = 50$ .

$k_j (j = 1, 2, 3)$	$E[C_T(n_D, \Delta t_{block}^*)]$
0.1	191599
1	191599
10	191599
100	191599

In this regards, the production of doctors requires a prudent planning from the viewpoint of economy.

Regarding the usefulness of the adaptive-penalty GA, a set of computations with fixed penalty parameters  $k_j (j = 1, 2, 3)$  is carried out. Each  $k_j$  is taken to be the same for each  $j$ . The optimization results from the fixed penalty for the case of  $N_{patient} = 50$  are shown in **Table 4**.

It should be noted that the optimization result from the adaptive-penalty GA for  $N_{patient} = 50$  is 187769. Therefore, it is obvious that the adaptive-penalty GA is superior to the fixed penalty GA. At the same time, **Table 4** also implies that the difficulty in selecting appropriate penalty parameters can be avoided when using the adaptive-penalty GA.

## 5. Conclusion

The appointment system belongs to the class of individual block/fixed interval. The adaptive-penalty GA has been applied in determining the optimal number of doctors and appointment interval. The length of the service time for a doctor consultation, the time required for the laboratory tests, and the time deviating from the appointment time are uncertain and modelled by random variables. The application of the proposed methodology has been shown through a numerical example. The numerical result has shown that the proposed methodology could effectively determine the optimal solution under a complicated objective function and the implicit constraints. Since the methodology provides the solutions under the consideration of uncertainty, the risk management of health care service can be realized.

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