Paper:

Cooperative Salvo Attack Using Guidance Law of Multiple Missiles

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In this article, a guidance problem for cooperative salvo attack of multiple missiles against a single stationary target is investigated. The proposed guidance law combines the well-known PNG law and cooperative acceleration command, which is based on the feedback of state error between the current missile and the mean value of participant missiles. The state variable in this paper is used as the approximate calculation of time-to-go. The cooperative acceleration command is designed to adjust the flight path and impact time, which leads the multi-missiles to hit the common target simultaneously. During the engagement, the velocities of missiles are not changed and presetting impact time is not needed. Simulation results show the effectiveness of the proposed guidance law.

Keywords: cooperative guidance, multi-missiles, proportional navigation, impact time, feedback

1. Introduction

Ever since the 1960s, efforts have been made to develop the accurate guidance of missiles, especially in the research of proportional navigation guidance (PNG) and optimal guidance. An optimal guidance law for a missile with an unconstrained acceleration vector was proposed by Hoetal in 1965 [1]. A comprehensive study of PNG was made by Guelman in 1971 [2], which is considered as a significant and fundamental theory of missile guidance. Furthermore, plenty of attention was paid to complement and extend the missile guidance theories.

As a matter of fact, spear is always accompanied by shield, such that more and more missile defense systems have been fielded to protect the target from enemy missiles. The most famous missile defense systems are National Missile Defense System (NMD) and Theatre Missile Defense System (TMD). The Close-in Weapon System (CIWS) is an important part of TMD, aiming at destroying incoming missiles at short range by closely grouped fire.

Warships and main battle tanks are mostly seen to be under protection of CIWS. These defensive weapons with powerful fire capability and various strategies seriously intimidate the survivability of the conventional missiles. Hence, missile developers have made great efforts to develop a high-performance missile system with ultimate sea-skimming flight and terminal evasive maneuvering capabilities despite a huge cost [3]. Usually, the shorter the engagement time, the better the missile survives the threats, as the reaction time of the target against the missile is reduced [4]. Also, missiles with terminal evasive maneuvering capability, to a certain extent, can avoid being shot during the attack course. Advanced missiles should make maximum speed, overload and thrust-to-weight ratio as large as possible to penetrate, and these missiles deserve expensive effort of control and cost much to produce. In this work, the strategy for cooperative attack is studied as another method for conventional missiles to survive and fulfill an attack task. This strategy requires the participant missiles to hit the target simultaneously. Clearly, it is difficult to defend against a group of attackers bursting into sight at the same time, even though each member is the conventional one in performance. So the simultaneous attack of multiple missiles is a cost-effective and efficient cooperative attack strategy taking advantage of the vulnerability of CIWS's one-by-one engagement feature [3].

Jeon et al. developed an impact-time-control guidance law (ITCG) for anti-ship missiles [4]. Jeon and Lee proposed a cooperative proportional navigation (CPN) for many-to-one engagements [3]. Both of the two guidance laws are based on the combination of PNG and the feedback of the impact time error. ITCG needs to preset a designated flight time to all missiles at the beginning of the homing phase, while all time-to-go (time left before hitting) should have been estimated in advance and the mean of time-to-go estimates of participant missiles is needed in the CPN. Sun and Xia proposed another cooperative guidance law which follows the conclusion in the work of Jeon and Lee and applies optimal guidance [5]. Zhao and Zhou introduced coordination algorithms into the guidance for multiple missiles [6]. Zou et al. proposed a distributed adaptive cooperative guidance law for multi-missiles with heterogeneous leader-follower structure to implement the cooperative salvo attacks [7]. The leader missile adopts a PN guidance law with constant navigation gain, while

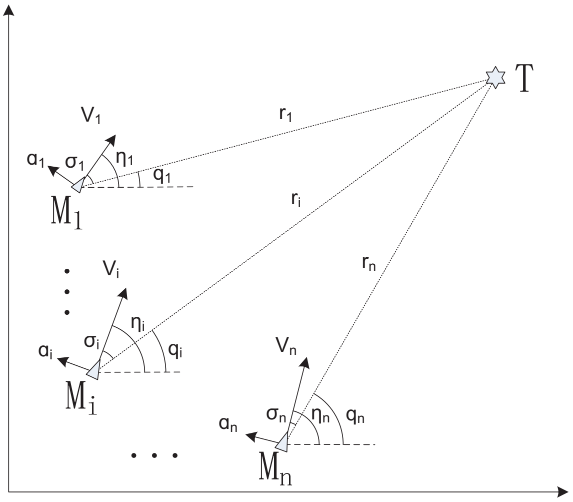

Fig. 1. Guidance geometry of salvo attack scenario.

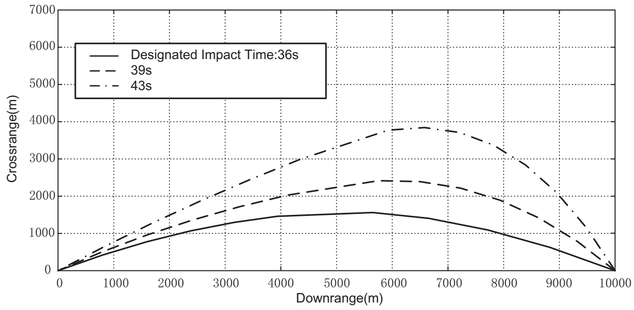

Fig. 2. ITCG trajectories with different designed impact time.

follower missiles adopt a distributed cooperative guidance law which is based on the PN guidance law with adaptive variable navigation gain. The research result of Zou et al. shows that speeds and missile-target distance of all the missiles converge to a compromise [7]. Sun et al. extended the work of Zou et al. for attacking maneuvering targets [8].

The goal of the cooperative guidance law proposed in this paper is to solve the salvo attack problem without changing missile speed, presetting impact time or having access to all estimates of the time-to-go of participant missiles. This article begins with introducing a statement of the homing problem for multiple missiles against a single stationary target. Next, the kinematic model and proposition of cooperative acceleration command are shown in Section 3. Then, simulation results illustrate the performance of the cooperative guidance in Section 4. Finally, conclusions are presented in Section 5.

2. Problem Statement

First of all, this paper mainly focuses on the guidance law of multiple missiles in cooperative attack. The missiles and the target are regarded as particles. Consider the homing guidance geometry of n missiles $M_1, M_2, \ldots, M_n$ salvo attacking a stationary target T, as shown in **Fig. 1**. Although each missile has a different missile-to-target range and an initial heading angle, they are requested to reach the target simultaneously. V_i denotes the speed of M_i and is supposed to be constant during the engagement. a_i is the acceleration command, which controls the missile and is perpendicular to missile velocity.

As illustrated, M_i and T are separated by a distance vector r_i, known as the line-of-sight (LOS). Flight-path angle and LOS angle are denoted by η_i and q_i, respectively, while σ_i stands for the heading error. η_i and q_i are positive when they are in the counterclockwise direction from reference line. σ_i is positive when the velocity vector is in the counterclockwise direction from LOS. In the homing process of a single missile against a stationary target, the guidance law is obligated to eliminate the heading error as soon as possible. Equally, the missile should not be too sensitive to disturbance. As shown in the well-known PNG

$$a = NV\dot{q}, \quad (1)$$

where N denotes the effective navigation constant and $\dot{q}$ is the rate of the line-of-sight angle. Then the flight-path angle rate is calculated as follows

$$\dot{\eta} = \frac{a}{V}, \quad (2)$$

or

$$\dot{\eta} = N\dot{q}. \quad (3)$$

The rate of LOS can be obtained as

$$\dot{q} = -\frac{V \sin\sigma}{r}, \quad (4)$$

and $\sigma = \eta - q$. Therefore, the governing equations can be expressed in terms of r and σ as follows

$$\dot{r} = -V\cos\sigma, \quad (5)$$

$$\dot{\sigma} = -\frac{(N-1)V\sin\sigma}{r}. \quad (6)$$

It is obvious from Eq. (6) that $|\sigma|$ would decrease to zero if $N > 1$. When σ is equal to zero, the missile will move along the LOS and straight toward the target, meanwhile the guidance command also converges to zero. N is chosen in the range from 3 to 5 in general cases, and mostly set to be 3 for energy-optimal control.

In order to attack the target simultaneously by multi-missiles without changing missile speed, a cooperative command is added to the PNG. The additional command is applied for adjusting the control effort by using the impact time differences with other missiles. This additional command has been proved efficient in ITCG [4]. As depicted in **Fig. 2**, the missile will generate a more bending trajectory when the designated impact time is longer than the original impact time. In the ITCG case, missile speed is not changed during the engagement. As the designated impact time is set longer, the optimal solution of the feedback problem will reduce the control effort (or even reverse the acceleration direction) in the early stage of the flight. Then the missile will follow a more bending and longer trajectory to hit the target at the designated time.

3. Cooperative Guidance Law

3.1. Kinematic Model

Consider the relative motion between missile M_i and stationary target T in polar coordinates:

$$\dot{r}_i = -V_i \cos\sigma_i$$
$$\dot{q}_i = -\frac{V_i \sin\sigma_i}{r_i},$$

for $\sigma_i = \eta_i - q_i$ and $\dot{\eta}_i = a_i/V_i$, such that

$$\dot{\sigma}_i = \frac{a_i}{V_i} + \frac{V_i \sin\sigma_i}{r_i}. \quad (7)$$

Let

$$\tau_i = \frac{r_i}{V_i \cos\sigma_i},$$

then the model turns into

$$\dot{\tau}_i = -1 + \tau_i \tan\sigma_i. \quad (8)$$

$$\dot{\sigma}_i = \frac{a_i}{V_i} + \frac{\tan\sigma_i}{\tau_i}. \quad (9)$$

3.2. Cooperative Acceleration Command

The acceleration command is composed of two different commands:

$$a_i = a_{Bi} + a_{Fi},$$

the first one is a_{Bi} for reducing the miss-distance, and the second one is an additional command a_{Fi} for synchronizing the impact time. In this paper, a_{Bi} is chosen as PNG command so that

$$a_{Bi} = NV_i\dot{q}_i.$$

The other command a_{Fi} is chosen as

$$a_{Fi} = sgn(\sigma_i)K\frac{V_i^2}{r_i}\varepsilon_i,$$

where K is a positive coefficient, ε_i is the difference between τ_i and the mean of τ_j ($j = 1\ldots m, j \neq i$).

$$\varepsilon_i = \left(\frac{1}{m-1}\sum_{j=1, j\neq i}^{m} \tau_j\right) - \tau_i. \quad (10)$$

Let

$$\bar{\tau} = \frac{1}{m}\sum_{j=1}^{m} \tau_j, \quad (11)$$

then

$$\varepsilon_i = \frac{m}{m-1}(\bar{\tau} - \tau_i). \quad (12)$$

Calculate the variance of τ as

$$\Sigma^2 = \frac{1}{m}\sum_{j=1}^{m} (\bar{\tau} - \tau_j)^2. \quad (13)$$

It will be shown that Σ^2 decreases during the homing guidance, when using the cooperative guidance law $a_i = NV_i\dot{q}_i + sgn(\sigma_i)K\frac{V_i^2}{r_i}\varepsilon_i$ with a positive gain K. The governing equations in term of r and σ can be expressed as

$$\dot{r}_i = -V_i\left(1 - \frac{\sigma_i^2(t)}{2}\right), \quad (14)$$

$$\dot{\sigma}_i = -\frac{(N-1)V_i\sigma_i}{r_i} + sgn(\sigma_i)\frac{V_i}{r_i}K\varepsilon_i. \quad (15)$$

Under the small angle assumption of σ_i

$$\sin\sigma_i = \sigma_i + O(\sigma_i^3), \quad (16)$$

$$\tan\sigma_i = \sigma_i + O(\sigma_i^3), \quad (17)$$

$$\cos\sigma_i = 1 - \frac{\sigma_i^2}{2} + O(\sigma_i^4). \quad (18)$$

From the differential equations, Eqs. (14) and (15), and Eq. (11), we get

$$r_i(t+\Delta t) = r_i(t) - V_i\Delta t\left(1 - \frac{\sigma_i^2}{2}\right), \quad (19)$$

$$\sigma_i(t+\Delta t) = \sigma_i(t) - \frac{V_i\Delta t[(N-1)\sigma_i(t) - Ksgn(\sigma_i(t))\varepsilon_i(t)]}{r_i(t)}. \quad (20)$$

Neglecting the higher-order terms of Δt, we have

$$\sigma_i^2(t+\Delta t) = \sigma_i^2(t) - 2\sigma_i^2(t)\frac{V_i\Delta t}{r_i(t)}\left(N - 1 - \frac{Ksgn(\sigma_i(t))\varepsilon_i(t)}{\sigma_i(t)}\right). \quad (21)$$

For

$$\tau_i(t) = \frac{r_i(t)}{V_i\cos\sigma_i(t)} \approx \frac{r_i(t)}{V_i}\left(1 + \frac{\sigma_i^2(t)}{2}\right),$$

such that

$$\tau_i(t+\Delta t) = \tau_i(t) - \Delta t - (N-1)\sigma_i^2(t)\Delta t + Ksgn(\sigma_i(t))\sigma_i(t)\varepsilon_i(t)\Delta t. \quad (22)$$

From Eq. (11), it can be derived that

$$\bar{\tau}(t+\Delta t) = \bar{\tau}(t) - \Delta t - \frac{(N-1)\Delta t}{m}\sum_{j=1}^{m}\sigma_j^2(t) + \frac{\Delta t}{m}\sum_{j=1}^{m} Ksgn(\sigma_j(t))\sigma_j(t)\varepsilon_j(t). \quad (23)$$

For

$$\Sigma^2(t+\Delta t) = \frac{1}{m}\sum_{j=1}^{m}\left(\bar{\tau}(t+\Delta t) - \tau_j(t+\Delta t)\right)^2, \quad (24)$$

such that

$$\Sigma^2(t+\Delta t) = \Sigma^2(t) - \frac{2}{m}\sum_{j=1}^{m}(\bar{\tau}(t) - \tau_j(t))[Ksgn(\sigma_j(t))\sigma_j(t)\varepsilon_j(t) - (N-1)\sigma_j^2(t)]\Delta t \quad . \ . \ . \ . \ . \ . \ . \quad (25)$$

where $\sigma_j(t)$ is supposed to be small, furthermore, $(N-1)\sigma_j^2(t)$ can be neglected in Eq. (25) if K is chosen to be relatively large.

Then consider

$$\begin{aligned}\Sigma^2(t+\Delta t) &= \Sigma^2(t) - \frac{2}{m}\sum_{j=1}^{m}(\bar{\tau}(t) - \tau_j(t))Ksgn(\sigma_j(t))\sigma_j(t)\varepsilon_j(t)\Delta t \\ &= \Sigma^2(t) - \frac{2}{m-1}\sum_{j=1}^{m}(\bar{\tau}(t) - \tau_j(t))^2 Ksgn(\sigma_j(t))\sigma_j(t)\Delta t\end{aligned}$$

$$. \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (26)$$

It is easy to see that $Ksgn(\sigma_j(t))\sigma_j(t)$ is always non-negative for a positive gain of K, such that the variance of $\tau_i(t)$ decreases. And in many articles, τ is used for the estimation of time-to-go, so the proposed cooperative guidance law $a_i = NV_i\dot{q}_i + sgn(\sigma_i)K\varepsilon_i V_i^2/r_i$ is effective in adjusting the impact time, especially at the beginning of the homing phase when the heading error is relatively larger. Note that when the missile is near the target, this command becomes PNG as heading error is zero. As the two parts of the acceleration command in this solution is perpendicular to missile velocity, engaged missiles will not decelerate for a longer flight time. The second part of the acceleration command ensures that this solution will synchronize the impact time of all missiles.

As the missiles approach the target, the control command may blow up because the sensitivity of a_{Fi} to the time-to-go error is inversely proportional to r_i. In order to circumvent this property, a switching rule is incorporated. The cooperative guidance law should be switched to PNG when the calculated variance of the impact time is reduced below a certain small value. This scheme could ensure a smooth homing process with acceptable impact time error.

4. Simulation Description

In this section, we illustrate the application of the proposed cooperative guidance law by considering a scenario in which three missiles attack a single stationary target. The target is located at $(0,0)$. The constant speeds, initial positions and headings of three missiles are shown in **Table 1**. Under the initial conditions, the initial heading errors σ_1, σ_2 and σ_3 can be calculated (see **Table 1**).

Table 1. Engagement scenario for cooperative attack.

Parameters	Missile 1	Missile 2	Missile 3
Velocity (m/s)	300	340	380
Initial Position (km)	(−6,6)	(−5,−8.66)	(5,−8.66)
Initial Heading (deg)	0	0	90
Initial Heading errors (deg)	45	−60	−30

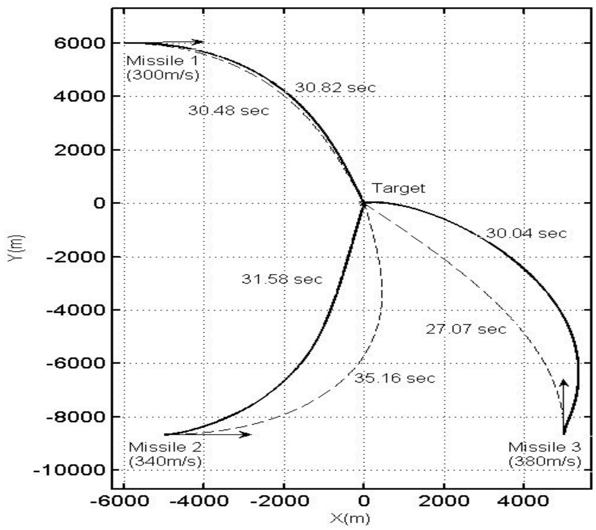

Fig. 3. Flight trajectories of three missiles.

Suppose that the navigation ratio N of three missiles is 3.2. As shown in **Fig. 3**, in the case of PNG, the impact time of the missiles are 30.46 s, 35.16 s and 27.07 s, respectively (trajectories are indicated by dashed lines).On the other hand, the proposed cooperative guidance law is used to drive the three missiles to hit the target simultaneously, with the same initial conditions (trajectories are indicated by solid lines). It is obvious from **Fig. 3** that all three missiles reach the destined point nearly at the same time. The discrepancy of impact time is less than 0.77 s around the mean value of time, so the intervals of arrival time are too short for the target to make effective response. The time histories of τ_1, τ_2 and τ_3 are depicted in **Fig. 4**, which figures out that the impact time of three missiles converges to a compromised value at the early stage of the process.

The variance of τ was proved to be decreasing during the cooperative homing phase in Section 3. **Fig. 5** shows the history of the variance of the impact time of three missiles. Due to the relatively large dispersion of impact time, the variance declines rapidly at the first 5 seconds.

In the simulations of Zou et al. [7], missiles adopting an adaptive guidance law decelerate sharply in the first 5 seconds of the adjusting time. In practice, however, a sharp deceleration leads to the risk of aerodynamic stall, reducing the feasibility of the guidance law. In contrast, the three missiles with the guidance law proposed in this pa-

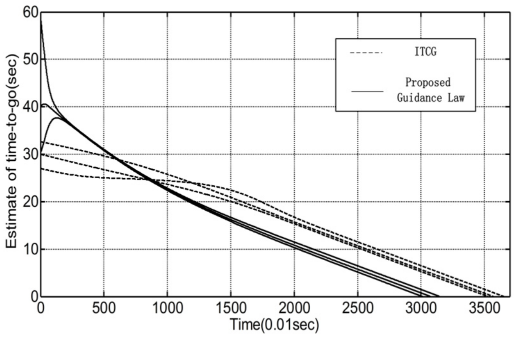

Fig. 4. Time histories of τ_i.

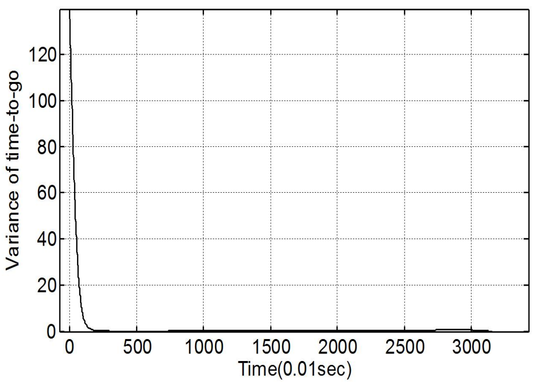

Fig. 5. Time histories of the variance of impact times.

per will attack the stationary target simultaneously without changing the speed of missiles.

Also, unlike the method proposed by Jeon et al. [4], the proposed guidance law does not need to prescribe any desired impact time before the homing phase starts. In the simulation of Jeon et al. [4], the impact time of four missiles with PNG are 35.67 sec, 30.83 sec, 27.40 sec, and 31.89 sec, respectively. Accordingly, they applied ITCG to the four missiles for simultaneous attack and designated impact time of 37 sec to the four missiles. As claimed in the work of Jeon et al. [4], the designated impact time should be selected to be larger than the maximum impact time of missiles by PNG. Instead, following the guidance law proposed in this paper, the time-to-go of the missiles during cooperative homing process converge to a certain mean value which is certainly smaller than the designated time in the research of Jeon et al. [4]. In this sense, the reaction time for the target to defense and survive will be reduced. Therefore, by the proposed guidance law, the simultaneous attack of multiple missiles can be completed more efficiently. The quantitative comparison between ITCG and proposed guidance law (under the same initial conditions) is shown in **Fig. 4** and **Table 2**. In **Fig. 4**, the starting values of two approaches are different because the estimate methods are different.

Noteworthy, it can be seen from **Figs. 3** and **4** that, the three missiles did not reach the target with exactly the same impact time, due to estimation error. Similar results were also observed in other works [4, 7]. The maximum difference among the impact time of all missiles is

Table 2. Flight time of ITCG and proposed guidance law.

Method	Missile 1	Missile 2	Missile 3
ITCG	35.27	35.40	36.22
Proposed guidance law	30.82	31.58	30.04

1.5 seconds, which is usually acceptable in practice. It was also observed that the heading errors of all missiles almost decrease to zero after 10 seconds, so the adjustment of is limited. Besides, **Fig. 5** demonstrates that the variance of the time-to-go of all missiles approaches zero very quickly. So, the missiles can reach consensus with acceptable performance.

5. Conclusion

The cooperative guidance problem for multi-missiles against a single stationary target is discussed. The governing equations expressed in terms of τ and σ were introduced, and the state variable τ was chosen as the estimate of impact time. Also, the proposed cooperative guidance law was proved to be effective. Furthermore, the law does not require any desired impact time be commanded to all missiles in advance. And, the missile speed was not changed during the adjustment, which is more feasible in application. Novel ways to diminish the impact time error would be studied to further improve the performance of the guidance law in the future.

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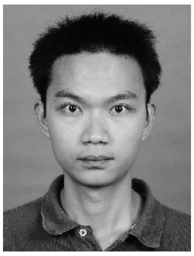

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