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Paper:

Evolution of Three Norms of Distributive Justice in an Extended Nash Demand Game

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The Nash demand game (NDG) has been at the center of attention when explaining moral norms of distributive justice on the basis of the game theory. This paper describes the demand-intensity game (D-I game), which adds an "intensity" dimension to NDG in order to discuss various scenarios for the evolution of norms concerning distributive justice, while keeping such simplicity that it can be analyzed by the concepts and tools of the game theory. We perform an ESS analysis and evolutionary simulations, followed by the analysis of replicator dynamics. It is shown that the three norms emerge: the one claiming an equal distribution (*Egalitarianism*), the one claiming the full amount (*Libertarianism*), and, as the special case of Libertarianism, the one claiming the full amount but conceding the resource in conflict (*Wimpy libertarianism*). The evolution of these norms strongly depends on the conflict cost parameter. Egalitarianism emerges with a larger conflict cost while Libertarianism with a smaller cost. Wimpy libertarianism emerges with a relatively larger conflict cost in libertarianism. The simulation results show that there are three types of evolutionary scenarios in general. We see in most of the trials the population straightforwardly converges to Libertarianism or Egalitarianism. It is also shown that, in some range of the conflict cost, the population nearly converges to Egalitarianism, which is followed by the convergence to Libertarianism. It is shown that this evolutionary transition depends on the quasi stability of Egalitarianism.

Keywords: social contract, distributive justice, nash demand game, evolutionary games

1. Introduction

Distributive justice concerns the distribution of socially valued goods and resources and the perceived fairness of outcomes that individuals receive. To operate successfully, a society needs to have a social norm on which to coordinate. Even in hunter-gatherer societies, successful hunters bargain with the other members of their tribe over the division of the preciously scarce meat. No boss is tolerated in societies that survived into modern times with a pure hunter-gathering economy and they share on a very egalitarian basis.

The Nash Demand Game (NDG) [1] has been widely employed to explain the emergence of moral norms, especially the evolutionary bases of distributive justice [2–8] as it encapsulates in the simplest way the situations in which obtained resource is divided among contributors. The NDG addresses the way moral norms coordinate expectations on how to divide the fruits of social cooperation [9]. The NDG is a one-shot two-player bargaining game. In this game, each player simultaneously demands a portion of some good. If the total amount demanded by the players is less or equal than available good, each player obtains the claimed request. Otherwise neither player gets anything.

Classical game theory is based on a normative theory of rational choice and prescribes what people ought rationally to choose ("normative approach"). The Nash equilibrium analysis is a normative approach. Recently, researchers in various fields have tried a different approach based on the evolutionary game theory that dispenses with strong assumption about rationality [10]. Rather than asking what moral norms ought to be, they aim at describing how people will in fact choose or how can the existing norms have evolved ("descriptive/evolutionary approach").

Nash equilibrium is the central concept especially in the normative approach. Every pair of the claimed demands that total 100% of the resources is a strict Nash equilibrium; however, people intuitively make the 50% demand [11, 12]. In his book, "Evolution and the Social Contract" [5], Skyrms proposed a pioneering study of evolution of fairness norms using the NDG. He provided a game theoretic account of how norms of fair division or justice might have evolved using replicator dynamics as a descriptive/explanatory approach, in which the evolutionary process might be biological or cultural (replication can be interpreted as imitation). With a 0.1 step size of the demand, a strategy asserting the half amount of resource is dominant in a group in the dynamics of 62% trials. Furthermore, with a finer step size and a higher probability of playing between similar strategies, the population always evolved into the fair division equilibrium. In succeeding studies, the assumption of correlated inter-

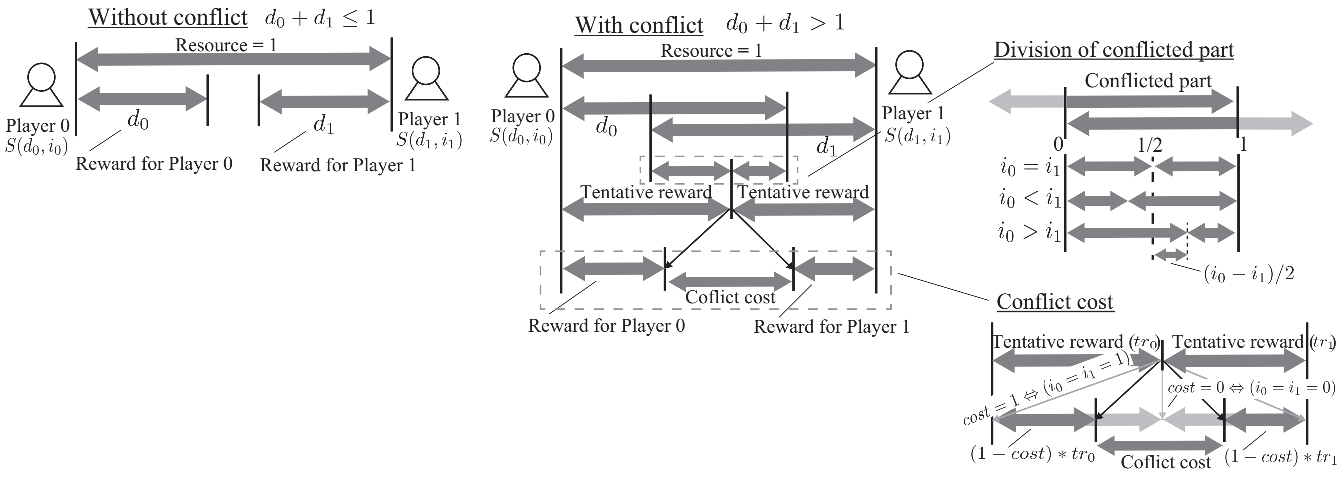

Fig. 1. Rewards in the D-I game.

actions of strategies has been criticized for the reason that it has no actual grounding in reality [13]. Instead, fair division has been achieved by spatial models in which interactions are limited to their neighborhood [2], and two-population models in which two players interact, one is sampled from a population and the other is from the other population [14].

Skyrmsian approach is evolutionary generalist as it entirely omits the psychological mechanisms, in contrast to evolutionary psychology, which emphasizes particular psychological factors of human behaviors [13]. Therefore, their models provide an abstract account of the evolution of the fairness norm by using the concepts and tools of the game theory. This paper deals with the demand-intensity game (D-I game), which adds an "intensity" dimension related to some psychological factor (e.g. bold or timid) to NDG in order to discuss various scenarios for the evolution of norms concerning distributive justice, while keeping such simplicity that it can be analyzed by the concepts and tools of the game theory [15]. This paper discusses the evolutionary dynamics of the D-I game, focusing on the three norms emerging in the game. To do so, we show evolutionary simulations in detail and consider replicator dynamics of the D-I game.

In NDG, if the sum of the demands exceeds the amount of resource, they obtain nothing[1]. This rigidity in evaluation of the conflict cost is weakened depending on the intensity values of the players. Therefore, the models based on the D-I game can describe various societies as follows. "People assert equality; if some people claim more than the equal amount of resource, the asserting people respect equality even though there is an associated high cost, e.g. a lawsuit." Otherwise, "people claim more than the equal amount against unselfish people; however, they surrender an extra demand to avoid a conflict."

2. D-I Game

As NDG, the D-I game is a two-player one-shot game and deals with the problem of allocating a limited resource between two players as shown in **Fig. 1**, in which d_0, i_0, d_1, and i_1 represent the self demand, the self intensity, the other's demand and the other's intensity, respectively.

Each player has a strategy $S(d,i)$ noted as a set of parameters d and i $(0 \leq d, i \leq 1)$. The parameter d represents the demand, which is a demanding amount in supposing a total amount of the resource is 1. If the total demand between the two players is not over 1 (the full amount of resource), each player gains the demand as a reward as in NDG.

Otherwise, firstly, the conflicted part of the resource $(d_0 + d_1 - 1)$ is divided according to the newly introduced parameter i, the intensity of the demand, as $1/2 + (i_0 - i_1)/2 : 1/2 + (i_1 - i_0)/2$. For example, $(i_0, i_1) = (0,0)$, $(0.5, 0.5)$, $(1, 0.5)$ or $(1, 0)$ makes the conflicted part divided as $1:1$, $1:1$, $0.75:0.25$ or $1:0$, respectively. Finally, each player gains the reward, which is calculated by reducing the tentative reward consisting of the divided conflicted part and the non-conflicted part, by a rate of the mean of i_0 and i_1. The larger the combined intensity between the players is, the smaller rewards both gain. If the sum is maximal $(i_0 = i_1 = 1)$ and the demands are conflicted $(d_0 + d_1 > 1)$, no reward is gained, as is the case with NDG. On the other hand, both players share the resource without a loss of a conflict when the combined intensity is minimal $(i_0 = i_1 = 0)$, even if the demands are conflicted. Therefore, the D-I game features a dilemma for both the demand and the intensity as follows: each player wants to receive more reward than the other and at the same time, wants to avoid the conflict cost.

The reward of player 0 (self) in the D-I game is defined

1. Nash himself proposed a smoothed version in which the probability that demands are met decreases rapidly to zero as the sum of demands exceeds 1 [16].

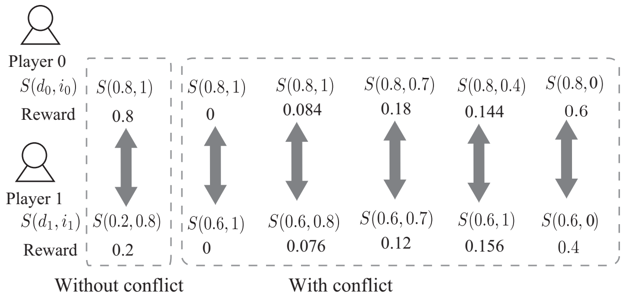

Fig. 2. Examples of the rewards in the D-I game ($r = 1$).

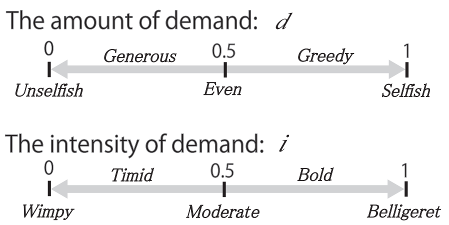

Fig. 3. Two dimensions of the strategy space for the D-I game.

as follows (**Fig. 1**).

$$Reward = \begin{cases} d_0 & (if\ d_0 + d_1 \leq 1) \\ tr \cdot (1 - cost) & (otherwise) \end{cases} \quad . \ . \quad (1)$$

$$tr = (1 - d_1) + (d_0 + d_1 - 1)\left(\frac{1}{2} + \frac{i_0 - i_1}{2}\right). \quad (2)$$

$$cost = \frac{i_0 + i_1}{2} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

tr is the tentative reward. The reward is reduced from tr by the conflict cost in the conflict case ($d_0 + d_1 > 1$). Examples of the rewards are shown in **Fig. 2**. Furthermore, we introduce a parameter r and use the following equation in place of Eq. (3) when considering various cost environments.

$$cost = \left(\frac{i_0 + i_1}{2}\right)^r \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

The parameter r in Eq. (4), a real number ranging from 0 to infinity, specifies the game structure in terms of the conflict cost. When $r = 0$, the conflict cost is maximized and the game is equivalent to NDG. A larger r corresponds to a lower conflict cost.

We refer to a strategy with demand d below 0.5 as "generous" and above 0.5 as "greedy" (**Fig. 3**). Specifically, the cases that d is 0, 0.5 and 1 are defined as "unselfish," "even" and "selfish," respectively. The parameter i, the intensity of the demand, represents how strong people claim their own demand in a conflict. The strategy is referred to as "timid" if $i < 0.5$, and as "bold" if $i > 0.5$. Specifically, the intensities i of 0, 0.5 and 1 are referred to as "wimpy," "moderate" and "belligerent, " respectively.

Regarding d, there are two typical strategies: $d = 0.5$ and $d = 1$. Although debatable, we simply associate the former with *Egalitarianism* and the latter with *Libertarianism*. When $r = 0$, all strategies with $d = 0.5$ are ESS (Evolutionary Stable Strategy), independently of i. Previous studies on a descriptive/evolutionary approach intended to describe how people could evolve this Egalitarianism [2, 5, 13, 14]. However, the strategies with $d = 0.5$ are not ESS except $r = 0$ and the norm becomes weaker as r increases. This is because a decrease of conflict cost (an increase of r) makes greed attractive.

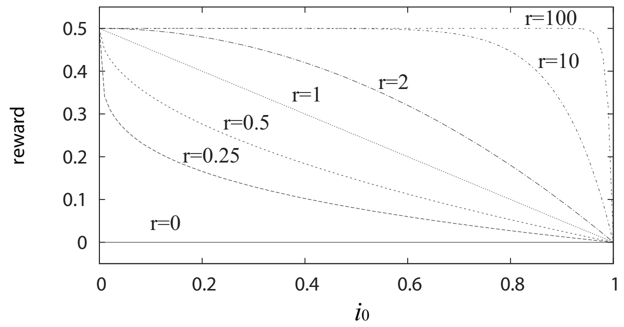

Fig. 4. Rewards for intensity i_0 when both players use the same greedy strategy.

When both players use the same greedy strategy ($d_0 = d_1 > 0.5$ and $i_0 = i_1$), the reward, which is $((1 - {i_0}^r)/2)$, is 0.5 at $i_0 = 0$ and decreases monotonically as i_0 increases for $0 < r < \infty$ (**Fig. 4**). A larger r weakens the decline. Thus, it is notable that *ideal* society in the sense of equality and efficiency can be achieved by not only a pure Egalitarianism ($S(0.5, *)$) but also an eventual equality norm based on Libertarianism ($S(1, 0)$) in the D-I game as shown later.

3. Game Theoretic Analyses

It is obvious that there are infinitely many Nash equilibria in the D-I game including $S(d_0, *)$ and $S(d_1, *)$ where $d_0 + d_1 = 1$. The ESS analysis leads to the detection of only three types of norms in these efficient strategies. The three norms (ESS) are *Egalitarianism* (norm A: $S(0.5, *)$, when $r = 0$), *Libertarianism* (norm B: $S(1, i^*)$, i^* depends on r as shown in **Fig. 5**, when $r \geq 0.5)^2$ and *Wimpy libertarianism* (norm C: $S(1, 0)$, when $0 < r < 0.773)^2$. **Fig. 5** shows the three norms in two dimensions of strategy: demand and intensity. **Fig. 6** shows the reward among these norms for $r = 0$, $r = 0.25$ and $r = 0.5$.

1. *Egalitarianism*
   Egalitarianism has an even property ($d = 0.5$). Egalitarianism exists only when $r = 0$ (NDG setting), which is represented as an ESS group of even strategies ($d = 0.5$) with any intensity value, as shown in **Fig. 5(a)**. Egalitarianism obtains the reward 0.5

2. These values are calculated from ESS equations.
$E(S,S) > E(T,S)$,
$E(S,S) = E(T,S)$ and $E(S,T) > E(T,T)$ for all T
S: own strategy, T: the other strategy, $E(A,B)$: the reward of A strategy playing with B.

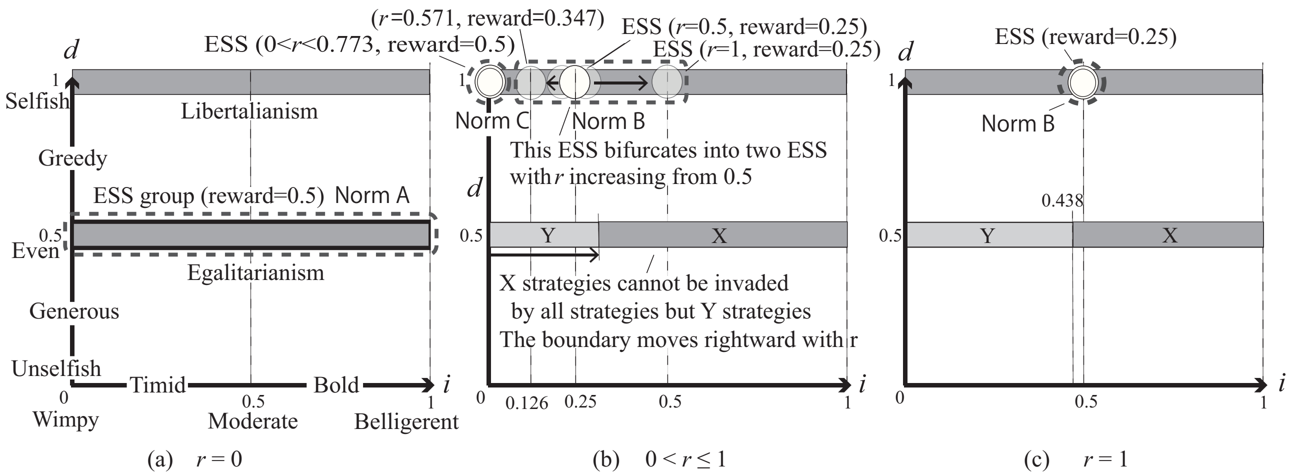

Fig. 5. Three types of norms in strategy space (Norm A: Egalitarianism, Norm B: Libertarianism, Norm C: Wimpy libertarianism). Panels (a)-(c) correspond to the cases of $r = 0$, $0 < r \leq 1$ and $r = 1$, respectively. The normal circles and a double circle in Panel (b) show norm B ($r \geq 0.5$) and norm C ($0 < r < 0.773$), respectively. The norm B is divided at $r = 0.5$ and the divided norms move to strategies with high and low intensities as shown in the shadow circles in Panel (b).

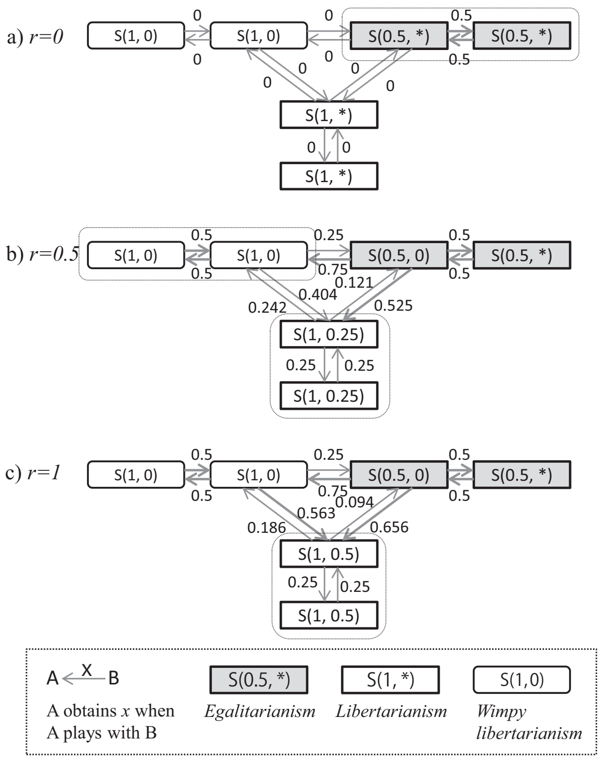

Fig. 6. Rewards playing among three types of norms.

among themselves.

2. *Libertarianism*
Libertarianism has a selfish property ($d = 1$). Libertarianism exists as an ESS under low-cost conditions ($r \geq 0.5$). Libertarianism bifurcates into two ESS at $r = 0.5$, $i = 0.25$. One of the bifurcated ESS becomes "more timid" ($0.5 \leq r < 0.571$)[2] along with increased r, and the other "bolder" ($r \geq 0.5$).

3. *Wimpy libertarianism*
Wimpy libertarianism also has a selfish property ($d = 1$). Wimpy libertarianism exists as an ESS under high-cost conditions ($0 < r < 0.773$). Wimpy libertarianism exists with the reward 0.5 among themselves.

It should be noted that a society with Wimpy libertarianism is ideal from the aspect of equality and efficiency as in the case with Egalitarianism, in the sense that each obtains the maximum reward (0.5) in the Wimpy libertarianism population. On the other hand, a society with Libertarianism is inefficient in the sense that each obtains less than 0.5 in the Libertarianism population. The reward in the libertarian population approaches 0.5 as r approaches infinity, in other words, the conflict cost approaches 0.

When $r > 0$, the egalitarian strategies are divided into two groups according to the value of the intensity (X and Y in **Figs. 5(b)**, **(c)**). The "bolder" egalitarian strategies (X) can be invaded only by all the egalitarian strategies (X+Y), while the "more timid" strategies (Y) can be invaded not only by all the egalitarian strategies (X+Y) but also by other strategies. Thus, the bolder egalitarian strategies have quasi stability. As r grows from 0 to 1, the range of bolder strategies (X) narrows from $0 \leq i \leq 1$ to $0.438 < i \leq 1$[3], and it finally vanishes when $r = 4.67$[3].

4. Evolutionary Simulations

We performed evolutionary simulations based on a genetic algorithm in order to understand the evolutionary scenarios concerning distributive justice, especially fo-

3. These values are calculated from ESS equations when own demand $d = 0.5$ and the other strategies (T in ESS equations) are all strategies except when $d = 0.5$.

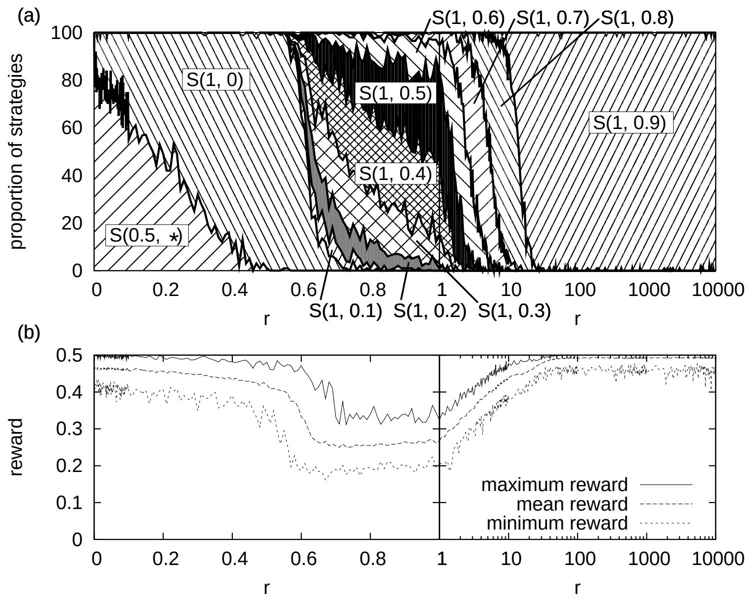

Fig. 7. (a) Proportion of the most common strategy in a population at the last generation over all 100 trials, for each r. (b) Average reward in the population. The graphs are the maximum, the mean and the minimum of the rewards over all 100 trials.

cusing on whether and how the three norms evolve that were identified in the ESS analysis. The population was composed of N individuals using the strategies d and i ranging form 0 to 1 in steps of S_d and S_i, respectively. The initial populations consisted of N individuals with randomly selected d and i. The fitness of each individual was defined as the total amount of rewards in playing the D-I game with all other members in the population. New individuals were generated by the three genetic operators: fitness-proportionate selection, crossover with the rate R_c, which simply exchanged the parent's intensity values, and mutation with the rate R_m, which randomly selected another value for d or i. We show here the results with $N = 100$, $S_d = S_i = 0.1$, $R_c = 0.5$, and $R_m = 0.05$.

Figure 7(a) shows the proportion of the 100 strategies, each of which dominated the population (was the most common) at the last (1500th) generation over all 100 trials, for each r. **Fig. 7(b)** shows the average reward in a population at the last generation.

For small r, the egalitarian strategies ($S(0.5,*)$) tended to occupy the population as the most common strategies. They prevailed at $r = 0$ with 100% of trials. Whereas the proportion over all 100 trials decreased along with a growth of r, the wimpy libertarian strategy became the most common strategies in more trials. The "wimpy" libertarian strategy $S(1,0)$ prevailed for $0 < r < 0.6$ as shown in **Fig. 7(a)**. For $0 \le r < 0.6$, an ideal society was achieved in the sense that all obtained nearly the successful reward (0.5) in the population of egalitarian strategies or the "wimpy" libertarian strategy. The most successful trial for $0 \le r < 0.6$ achieved the reward of around 0.5 ("maximum reward" in **Fig. 7(b)**).

For $0.6 < r < 10$, other libertarian strategies ($d = 1$) tended to become the most common strategies. They were various for each trial, although we see a low reward in the population. The intensity of the evolved libertarian strategies became larger as r increased. Those various strategies include not only the libertarian ESS but also other libertarian strategies. We observed that libertarian strategies coexisted with other libertarian strategies and populations fluctuated between them (**Fig. 8(d)**). It might be due to a little difference in the rewards between similar libertarian strategies.

For $r > 1$, high intensity strategies in libertarian strategies ($S(1,0.5) \sim S(1,0.9)$) prevailed in the population and the reward in the population became larger as r increased. This is simply because the reduction of the conflict cost strongly affects the reward. An ideal society is also achieved when r approaches infinity (no conflict cost).

We found that there were three types of evolutionary scenarios (**Fig. 9**): evolution straightforwardly converging to an egalitarian population, evolution converging to a libertarian population through an egalitarian population, evolution straightforwardly converging to a libertarian population. In the first and third types of scenarios, a population quickly converged to an egalitarian or a libertarian population (**Figs. 9(a)** and **(c)**, respectively). The frequency of the latter (former) scenario increased (decreased) as r increased, i.e. the conflict cost decreased.

We observed the second type of scenarios (**Fig. 9(b)**) in fewer trials (e.g. 9% for $r = 0.2$). **Fig. 8(a)** shows a typical evolution of the second type, in which the evolutionary transitions of the average intensity and the average reward in a population for $r = 0.2$ are shown. We see that Egalitarian strategies quickly prevailed and remained, and then libertarian strategies invaded after certain generation around 600th generation. In this evolution, the "bolder" egalitarian strategies (X in **Fig. 5**) predominated at the initial generation. They have quasi stability

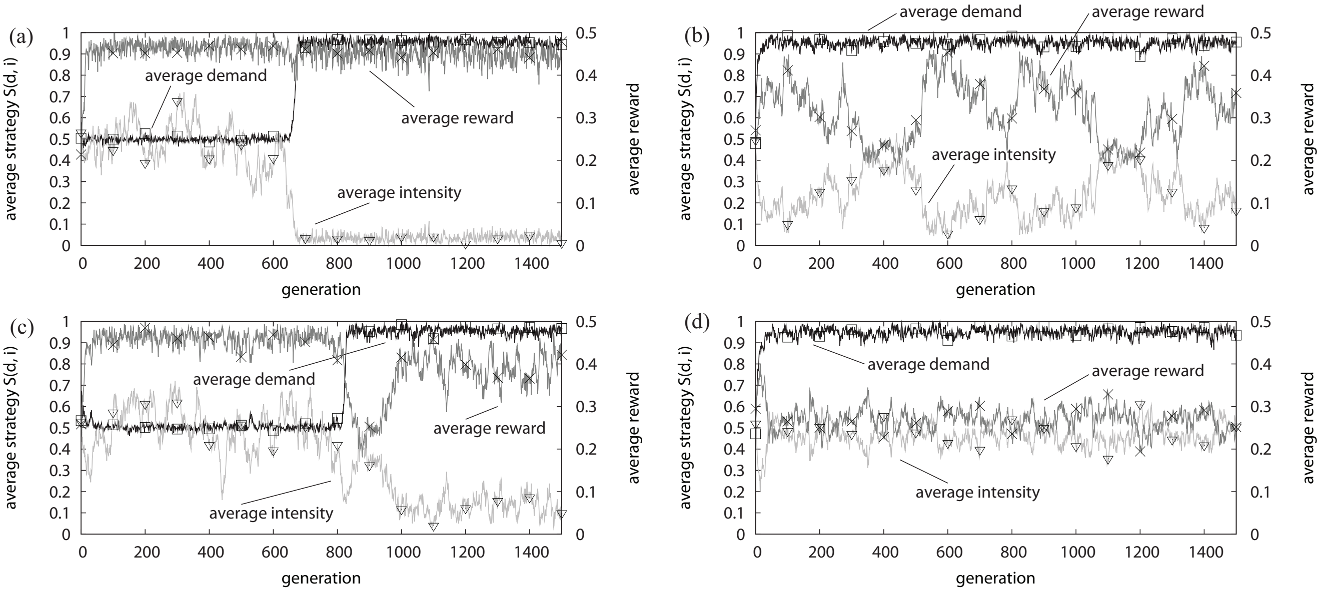

Fig. 8. Average demand, average intensity and average reward in a population through an evolutionary simulation for $r = 0.2$ (a), $r = 0.6$ (b), (c) and $r = 1$ (d).

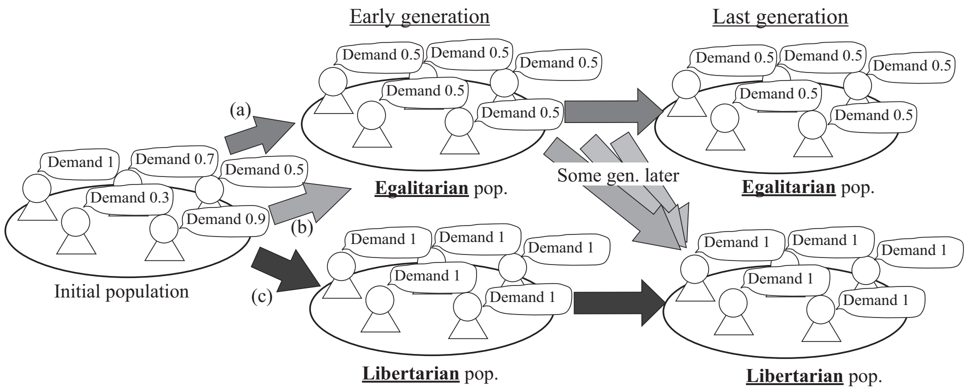

Fig. 9. (a) Evolution into an **egalitarian** population. (b) Evolution into a **libertarian** population through an **egalitarian** population. (c) Evolution into a **libertarian** population.

as discussed in the previous section. Then, the population has an evolutionary stability against the invasion by libertarian strategies. However, the "more timid" egalitarian strategies (Y in **Fig. 5**) easily invaded the population by the mutation operator. The strategies made it easier for libertarian strategies to prevail in the population.

In contrast, we never observed the evolutionary transitions from the population occupied with the libertarian strategies to the population with egalitarian strategies. Furthermore, we never observed the coexistence between egalitarian strategies and libertarian strategies. Here we consider the reason. A libertarian strategy obtains three times as much as an egalitarian strategy when they play the game with each other under the same intensity. The difference is larger when i is smaller. Once libertarian strategies, especially the wimpy libertarian strategy, invade in a population, egalitarian strategies obtain the lower reward and cannot remain in the population. In addition, for "stable" libertarian strategies there is no strategy which made it easier for egalitarian strategies to prevail in the population.

Furthermore, in the third type, we found that there were two types of intensity evolutions during the evolutionary transition from egalitarian to libertarian populations: one is with alterations between wimpy and moderate strategies, and the other is with fluctuations between similar strategies. **Fig. 8(b)** corresponds to the first case. For around $r = 0.6$, the demand of a population quickly converged to the selfish strategy ($d = 1$) and then the population alternated between some libertarian strategies ($S(1, 0.1) \sim S(1, 0.4)$) and the wimpy libertarian strategy. **Fig. 8(d)** corresponds to the latter case, in which there were the fluctuations between similar strategies ($S(1, 0.4) \sim S(1, 0.6)$). This might be because these similar strategies obtain similar rewards between them.

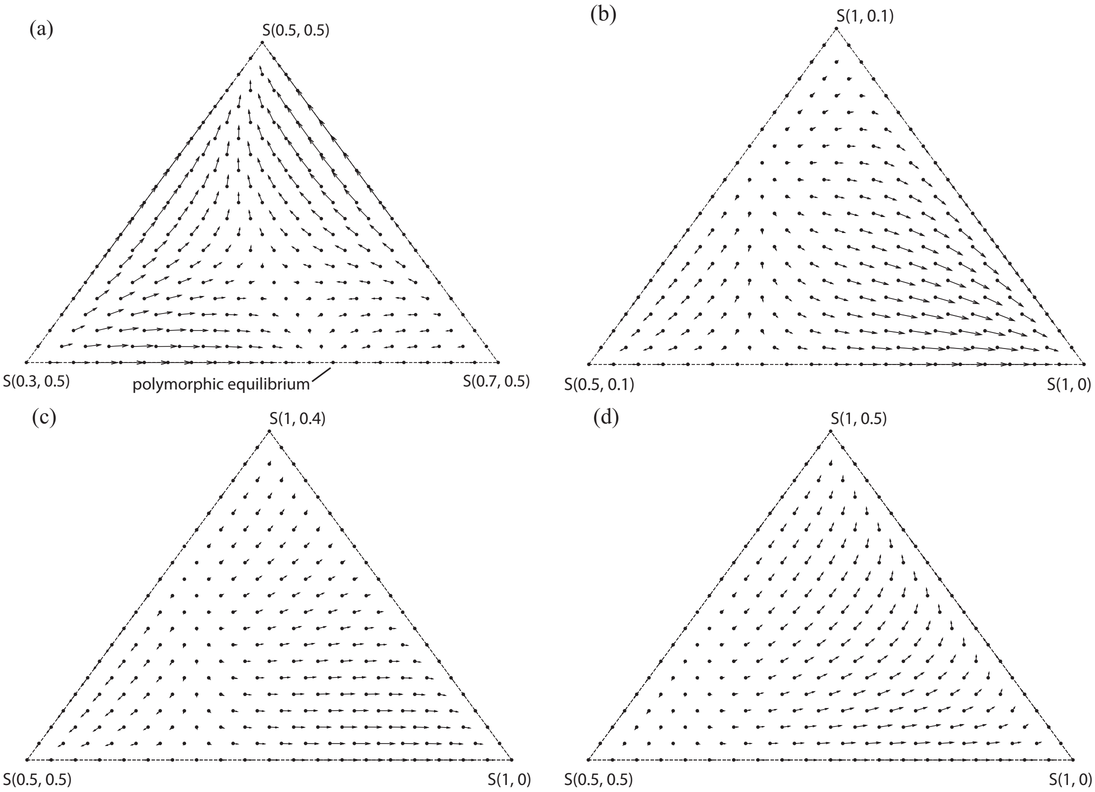

Fig. 10. Replicator dynamics. (a) populations playing $S(0.3, 0.5)$, $S(0.7, 0.5)$ and $S(0.5, 0.5)$ for $r = 0.2$. (b) populations playing $S(0.5, 0.1)$, $S(1, 0)$ and $S(1, 0.1)$ for $r = 0.2$. (c) populations playing $S(0.5, 0.5)$, $S(1, 0)$ and $S(1, 0.4)$ for $r = 0.6$. (d) populations playing $S(0.5, 0.5)$, $S(1, 0)$ and $S(1, 0.5)$ for $r = 1$.

5. Replicator Dynamics

Here we focus on two types of interesting evolutionary scenarios: egalitarian populations changed to libertarian populations, and the populations alternate between libertarian and wimpy libertarian populations. We further clarify the evolutionary dynamics of these evolution, using the following difference equation describing the replicator dynamics.

$$x_s(t+1) - x_s(t) = x_s(t) \cdot \{f_s(t) - \bar{f}(t)\} \quad . \; . \; (5)$$

x_s is the proportion playing strategy s in a population and $f_s(t)$ is the fitness of strategy s at t-th step. $\bar{f}(t)$ is the mean of the fitness. The fitness is defined as an average reward when playing the D-I game with the other members in the population.

This paper visualizes the replicator dynamics as a vector field by taking up three typical strategies (**Fig. 10**). Each lattice position in the triangles of the figure corresponds to a composition of a population playing these three strategies. The lattice scale indicates 5% of the difference of the population composition. Each vertex of the triangle represents 100% of the population playing the corresponding strategy. Each head of a vector expresses the population composition playing the strategies in the next step under the replicator dynamics. Thus, each vector expresses the strength and direction of selection and these figures show what sort of initial population compositions leads to what kind of equilibria. The resilience of each equilibria state is measured by the size of their respective basins of attraction – the areas from which the evolutionary dynamics leads to them.

Skyrms studied an egalitarian and a polymorphic equilibria by using a vector filed of the replicator dynamics of the populations playing $S(1/3, *)$, $S(2/3, *)$ and the egalitarian strategy $S(1/2, *)$ in NDG [5]. First, we examined the equilibria in the D-I game settings ($r > 0$) by using the replicator dynamics of the populations playing $S(0.3, 0.5)$, $S(0.7, 0.5)$ and an egalitarian strategy $S(0.5, 0.5)$. **Fig. 10(a)** shows the replicator dynamics of populations playing these strategies. We see a similar dynamics as in the case studied by Skyrms. It is shown that there are two basins of attraction, one converging to a division of the whole population between $S(0.3, 0.5)$ and $S(0.7, 0.5)$, and one converging to the universality of $S(0.5, 0.5)$. As r increases, the polymorphic equilibrium approaches the universality of $S(0.7, 0.5)$.

Next, we show the replicator dynamics of the populations playing each of the strategies that can be ESS norms: *Egalitarianism*, *Libertarianism* and *Wimpy libertarianism* in **Figs. 10(b)**, **(c)**, and **(d)**. **Fig. 10(b)** shows the replicator dynamics of populations playing $S(0.5, 0.1)$, $S(1, 0)$, and $S(1, 0.1)$. We observe that there are two basins of attraction, one of the egalitarian equilibrium ($S(0.5, 0.1)$)

and one of the wimpy libertarian equilibrium ($S(1,0.1)$). The most of the population compositions evolve into the latter equilibrium while the former attracts some populations. **Fig. 10(c)** shows the case with the populations playing $S(0.5,0.5)$, $S(1,0)$, and $S(1,0.4)$. There are three moderately balanced basins of attraction, one of the egalitarian equilibrium, one of the libertarian equilibrium and one of the wimpy libertarian equilibrium. **Fig. 10(d)** shows the case with the populations playing $S(0.5,0.5)$, $S(1,0)$, and $S(1,0.5)$. In this case, we observed a basin of attraction of the egalitarian equilibrium and that of the libertarian equilibrium. The basin of the egalitarian equilibrium is small and most of the population compositions evolve to the population all playing libertarian strategy.

In the rest of this section, we will compare the above results with the evolutionary simulations described in the previous section. In most of the simulations, we observed that the population quickly evolved not to the population with a polymorphic equilibrium but to the egalitarian or libertarian population. This might be due to the fact that the attraction basin of the populations playing polymorphic equilibria is relatively small compared to the one of the universality of the egalitarian or libertarian strategies as shown in **Fig. 10(a)**.

Evolutionary simulations also showed that not only when $r = 0$ but even when $r > 0$, the population could evolve to the one occupied with the egalitarian strategies although the egalitarian strategies for $r > 0$ are not ESS. **Fig. 10(b)** shows a significantly large basin of attraction for even $S(0.5,0.1)$, which is rather a weak egalitarian strategy having a quasi stability, for $r = 0.2$ along with a basin of attraction of the wimpy libertarian strategy $S(1,0)$. This seems to cause the population to evolve the egalitarian or wimpy libertarian strategies unlike the above case with the polymorphic equilibria having a smaller size of basin of attraction.

We also found in the evolutionary simulations that in some range of the conflict cost (e.g. $r = 0.2$), the population nearly converged to the egalitarian strategies, which was followed by the convergence to the libertarian strategies. On the contrary, we never observed the evolutionary transitions from the population occupied with the libertarian strategies to the one with egalitarian strategies. These can be attributed to the larger basin of attraction of the wimpy libertarian strategy than that of the egalitarian strategies shown in **Fig. 10(b)**.

In the evolutionary simulations with $r = 0.6$, we observed that a population alternated between some libertarian strategies and the wimpy libertarian strategy. This can be due to the fact that both substantial basins of attraction of the libertarian strategy $S(1,0.4)$ and the wimpy libertarian strategy $(1,0)$ are equally large as shown in **Fig. 10(c)**.

Figure 10(c) shows that there is a considerable-sized basin of attraction of egalitarian strategies along with these basins. This induced the evolution shown in **Fig. 8(c)** in which the population first evolved the egalitarian strategies, which was followed by the alteration between the libertarian and the wimpy libertarian strategies.

Finally, in the case with a large conflict cost (e.g. $r = 1$), it converged only to the libertarian strategies (not to the wimpy libertarian strategy). We might see the reason in **Fig. 10(d)**. The basin of attraction of the wimpy libertarian strategy does not exist, while that of egalitarian strategies still exist but with a smaller size. This might be the reason why the egalitarian strategies and the wimpy libertarian strategies could not prevail for $r = 1$ in the evolutionary simulations.

6. Conclusion

This paper discussed the possible scenarios for the evolution of norms concerning distributive justice using the D-I game, which adds an "intensity" dimension to the Nash Demand Game. We did game theoretic analyses of the D-I game and performed evolutionary simulations.

ESS analysis and evolutionary simulations showed the evolution of three types of norms: *Egalitarianism*, *Libertarianism*, and *Wimpy libertarianism*. While Wimpy libertarianism is classified into Libertarianism as it claims the full resource, it can also achieve an egalitarian division as a result in a population without conflict cost. It was shown that the level of conflict cost has a large influence on which norm emerges. Egalitarianism emerges with a larger conflict cost while Libertarianism with a smaller cost. Wimpy libertarianism emerges with a relatively larger conflict cost in libertarianism.

The simulation results show that there are three types of the evolutionary scenarios in general. We see in most of the trials the population straightforwardly converges to Libertarianism or Egalitarianism. It is also shown that, in some range of the conflict cost, the population nearly converges to Egalitarianism, which is followed by the convergence to Libertarianism. It is shown that this evolutionary transition depends on the quasi stability of Egalitarianism.

We showed that quasi stability of egalitarian strategies played an important role in the evolution. According to our ESS analysis, the bolder egalitarian strategies can be invaded only by any egalitarian strategies, while the more timid egalitarian strategies can be invaded not only by all the egalitarian strategies but also by other strategies. The quasi stability of egalitarian strategies comes from this property of the bolder egalitarian strategies. The replicator dynamics analysis revealed the existence of a basin of attraction of the strategies with a considerable size, which would support the stable property of them.

The conflicting cost can be interpreted as a psychological cost. We may not feel the conflict cost as a pressure when deciding how to share sweets among family members. We might strongly demand the whole sweets like a libertarian. However, when the sharing members are friends, we may feel a pressure more and act like a wimpy libertarian. Furthermore, if the shared resource is money, we may feel seriously the cost as a pressure than the case with sweats and make an equal demand strictly as an egalitarian.

Our future work includes the comparison of the obtained theoretical results with the results of the cultural evolution experiments using human subjects. The preliminary experiment we have conducted showed that introducing an adequate metaphor to incorporate a psychological factor as the intensity dimension in the experiment has a great impact on the emergence of the norms [17]. We believe that the D-I game will provide us with a useful framework to study dynamics of distributive justice from an emergence perspective, beyond the conventional question of whether strategies demanding equal share can dominate the population.

References:

[1] J. F. Nash, "The bargaining problem," Econometrica, Vol.18, pp. 155-162, 1950.

[2] J. M. Alexander, "Evolutoinary explanations of distributive justice," Philosophy of Science, Vol.67, No.3, pp. 490-516, 2000.

[3] K. Binmore, "Game Theory and the Social Contract, Vol.2: Just Playing," The MIT Press, Cambridge, 1998.

[4] A. Rubinstein, "Perfect Equilibrium in a Bargaining Model," Econometrica, Vol.50, pp. 97-109, 1982.

[5] B. Skyrms, "Evolution of the Social Contract," Cambridge University Press, Cambridge, 1996.

[6] R. Sugden, "The Economics of Rights, Co-operation and Welfare," Basil Blackwell, Oxford, 1986.

[7] P. H. Young, "An Evolutionary Model of Bargaining," J. of Economic Theory, Vol.59, No.1, pp. 145-168, 1993.

[8] P. H. Young, "Individual Strategy and Social Structure: An Evolutionary Theory," Princeton University Press, Princeton, 1998.

[9] A. Andreozzi, "An evolutionary theory of social justice: Choosing the right game," European J. of Political Economy, Vol.26, No.3, pp. 320-329, 2010.

[10] G. Mar, "Evolutionary game theory, morality, and darwinism," D. Katz (ed.), Evolutionary Origins of Morality, pp. 322-326, Imprint Academic, 2000.

[11] C. T. Dawes, J. H. Fowler, T. Johnson, R. McElreath, and O. Smirnov, "Egalitarian motives in humans," Nature, Vol.446, pp. 794-796, 2007.

[12] M. Hsu, C. Anen, and S. R. Quartz, "The right and the good: dstributive justice and neural encoding of equity and efficency," Science, Vol.320, pp. 1092-1095, 2008.

[13] J. D'Arms, R. Batterman, and K. Górny, "Game theoretic explanations and the evolution of justice," Philosophy of Science, Vol.65, No.1, pp. 76-102, 1998.

[14] H. Ohtsuki, "Evolutionary Dynamics of the Nash Demand Game: A Diffusion Approach," Dynamic Games and Applications, Vol.1, pp. 449-461, 2011.

[15] K. Kojima and T. Arita, "How do equity norms evolve? - An evolutionary game theory approach to distributive justice," Artificial Life and Robotics, Vol.17, No.2, pp. 287-292, 2012.

[16] J. F. Nash, "Two-person Cooperative Games," Econometrica, Vol.21, pp. 128-140, 1953.

[17] S. Nakamura, K. Kojima, R. Suzuki, and T. Arita, "Design of Experiments on a Game Focused on Distributive Justice Using Human Subjects," Proc. of the SICE 39th Intelligent System Symp., pp. 271-276, 2012 (in Japanese).

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