**Paper:**

# Square Layout Four-Point Method for Two-Dimensional Profile Measurement and Self-Calibration Method of Zero-Adjustment Error

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**An on-machine measurement method, called the square-layout four-point (SLFP) method with angle compensation, for evaluating two-dimensional (2-D) profiles of flat machined surfaces is proposed. In this method, four displacement sensors are arranged in a square and mounted to the scanning table of a 2-D stage. For measuring the 2-D profile of a target plane, height data corresponding to all measuring points are acquired by means of the raster scanning motion. At the same time, pitching data of the first primary scan line and rolling data of the first subsidiary scan line are monitored by means of two auto-collimators to compensate for major profile errors that arise out of the posture error. Use of the SLFP method facilitates connection of the results of straightness-measurements results obtained for each scanning line by using two additional sensors and rolling data of the first subsidiary scan line. Specifically, the height of a measuring point is calculated by means of a recurrence equation using three predetermined height data for adjacent points in conjunction with data acquired by the four displacement sensors. Results of the numerical simulation performed in this study demonstrate higher efficiency of the SLFP method with angle compensation. During actual measurement, however, it is difficult to perfectly align inline the origin height of each displacement sensor. With regard to the SLFP method, zero-adjustment error is defined as the relative height of a sensor's origin with respect to the plane comprising origins of the other three sensors. This error accumulates in proportion to number of times the recurrence equation is applied. Simulation results containing the zero-adjustment error demonstrate that accumulation of the said error results in unignorable distortion of measurement results. Therefore, a new self-calibration method for the zero-adjustment error has been proposed. During 2-D profile measurement, two different calculation paths – the raster scan path and orthogonal path – can be used to determine the height of a measurement point. Although heights determined through use of the two paths must ideally be equal, they are observed to be different because accumulated zero-adjustment errors for the two paths are different. In view of this result, the zero-adjustment error can be calculated backwards and calibrated. Validity of the calibration method has been confirmed via simulations and experiments.**

**Keywords:** on-machine measurement, multi-point method, profile measurement, flatness, software datum

## 1. Introduction

High-precision straight bars and flat plates are used as linear and planer guides at different stages of various machine tools.

Manufacture of high-precision straight bars and flat plates requires compensation machining and finishing based on measurement data for feedback. Accuracy of profile measurement is, therefore, essential as it tends to limit accuracy of the machining process.

A combination of displacement sensor(s) and a hardware reference, such as the straightedge or master flat plane, is used for measuring profiles of straight bars and flat planes [1]. This method is widely employed, but reference deformation under the action of gravity and other factors affects the accuracy of measurement results. In particular, large-sized references demonstrate noticeable deformation and are difficult to handle.

Therefore, on-machine measurement methods without using hardware references are required for measurement of large-scale flat surfaces, and in such a method, elimination of motion errors during the scanning is essential.

The laser autocollimation method has been developed for measuring the profile of a near-flat mirror surface [2, 3]. This method and its variations are free from translational errors along the $Z$-direction as well as the pitching error in principle.

As versatile methods, many error-separation techniques, referred to as software datum [4], have been devel-

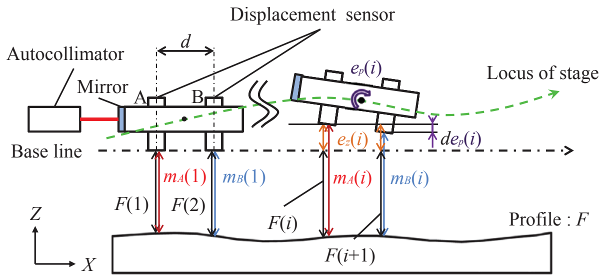



**Fig. 1.** Two point method with angle compensation.

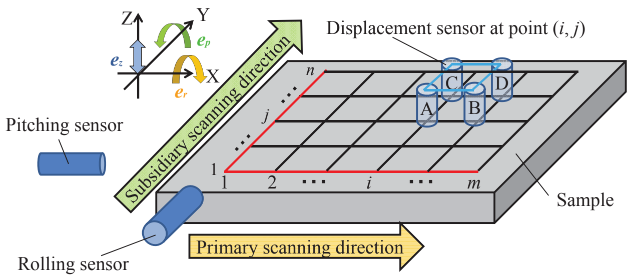



**Fig. 2.** Schematic of SLFP method with angle compensation.

oped to reconstruct a surface profile by separating motion errors of a stage from measurement data of multiple scans and/or multiple sensors [5, 6].

For measuring one-dimensional (1-D) profiles, many error-separation techniques employing multiple sensors, two-point methods [7–13], three-point method [8, 14–16], four-point method [8, 17], and multi-point method [18] have been developed.

In contrast, two-dimensional (2-D) profile-measurement techniques [19, 20] have not attracted much attention, since it is considered that 1-D profile measurements could be performed along two perpendicular axes by combining the mathematical data-connection processes or through use of two large bar mirrors. However, data-connection methods are not yet well-established, and large bar mirrors are difficult to maintain in a general machine shop. The authors, therefore, propose a practical 2-D profile-measurement method, called the "square-layout four-point method" (SLFP method) with angle compensation. Furthermore, to reduce accumulation of the zero-adjustment error, a self-calibration method has also been proposed.

## 2. Principles of the Two-Point and Square-Layout Four-Point Methods

### 2.1. Two-Point Method for Straightness Measurement

The sequential two-point method and its variations are well-known as scanning-measurement methods for evaluating 1-D profiles of a near-flat surface.

**Figure 1** depicts a schematic of the sequential two-point method with angle compensation. Two displacement sensors – A and B – placed a distance $d$ apart, are mounted on a scanning table. A target mirror is placed on the left of the table, and the pitching error $e_p$ is monitored by means of an autocollimator. The table moves along the $X$-axis, and data measured by displacement sensors and the autocollimator were captured with pitch $d$, which correspond to an interval of two displacement sensors. Outputs $m_A$ and $m_B$ from sensors A and B could be expressed as follows.

$$m_A(i) = F(i) + e_z(i) \quad \quad (1)$$

$$m_B(i) = F(i+1) + e_z(i) + de_p(i) \quad \quad (2)$$

where $F(x)$ denotes height of the surface being tested, and $e_z$ denotes the translational error of the stage.

By subtracting Eq. (1) from Eq. (2), $e_z$ can be eliminated, and a recurrence equation in terms of $F(x)$ can be deduced.

$$F(i+1) = F(i) + m_B(i) - m_A(i) - de_p(i) \quad \quad (3)$$

During actual measurements, there exist two error sources. One is the zero-adjustment error $\delta$, which arises from the difference between origin heights of sensors A and B. The other is the table inclination at the start position. Both error sources appear in the form of baseling inclination components in the final 1-D profile-measurement result. During straightness measurement, these errors do not cause so much concern, since they exist as linear components and can easily be compensated for.

During 2-D measurements, however, these errors arise out of the difference between datum lines of the measured profile for each line. Consequently, 2-D profile of the flat surface cannot be reconstructed by simply arranging 1-D profiles of each line. Therefore, measured data for each line have to be connected using other data that relate the relative height and inclination of each datum line.

### 2.2. Square-Layout Four-Point Method for Flatness Measurement

To solve the problem of 2-D profile measurement, few four-point methods have previously been proposed. These methods, however, appeared to be restricted in some way or required complex calculations. This study proposes the simple yet practical square-layout four-point method with angle compensation.

In this method, four displacement sensors – A–D – are mounted on a scanning table and arranged to form a square with each side measuring $d$, as depicted in **Fig. 2**. The $X$-axis refers to the primary scanning direction while the $Y$-axis forms the subsidiary scanning direction. Intervals of scanning motions along the $X$- and $Y$-axes must correspond to intervals between sensors to meet the requirements of a sequential scanning method. Position of the table is indicated by the position of sensor A. Pitching $e_p$ and rolling $e_r$ motions are defined with respect to the primary scan.

The measuring procedure could be described as follows. The primary scan is first performed between points $(1,1)$ and $(m-1,1)$, and profile data along $X$-axis are acquired in combination by sensors A and B. Simultaneously, during the same scan, relative heights of the two measuring points on the next line are acquired by sensors C and D. At this time, pitching motion – $e_p(i,1)$ – of the table is also monitored by means of an angle sensor installed near the start point on the primary scan line.

At the end of the $i$-th scan line $(m-1,i)$ during the primary scan $(i \geq 1)$, the stage table returns to the start point $(1,i)$. Subsequently, the table shifts along the $Y$-axis to perform the subsidiary scan and moves to point $(1,i+1)$. Simultaneously, the rolling motion $e_r(1,j)$ during the subsidiary scan is monitored by means of another angle sensor located near the start point on the subsidiary scan line.

The $(i+1)$-th primary scan was then performed. On the secondary and following lines, outputs of only the four displacement sensors are acquired sans the need for angle monitorings, because pitching and rolling motions monitored during the primary scan could be corrected when the recurrence in Eq. (8), described below, was applied.

The above procedure was repeated until the last $(n-1)$-th line.

Outputs $m_A$–$m_D$ obtained from sensors A–D, could be described as follows.

$$m_A(i,j) = F(i,j) + e_z(i,j) \quad \ldots \quad (4)$$

$$m_B(i,j) = F(i+1,j) + e_z(i,j) + de_p(i,j) \quad \ldots \quad (5)$$

$$m_C(i,j) = F(i,j+1) + e_z(i,j) + de_r(i,j) \quad \ldots \quad (6)$$

$$\begin{aligned} m_D(i,j) &= F(i+1,j+1) + e_z(i,j) \\ &\quad + de_p(i,j) + de_r(i,j) \quad \ldots \quad (7) \end{aligned}$$

where $F(i,j)$ denotes height of a point $(i,j)$ on the surface being tested; $e_z(i,j)$ denotes translational error of the table; $e_p(i,j)$ and $e_r(i,j)$ denote pitching and rolling motions, respectively, of the table at position $(i,j)$.

During the profile reconstruction process, Eqs. (4)–(7) are solved as simultaneous equations, and the following recurrence formula is deduced.

$$\begin{aligned} F(i+1,j+1) &= m_A(i,j) - m_B(i,j) - m_C(i,j) \\ &\quad + m_D(i,j) - F(i,j) \\ &\quad + F(i+1,j) + F(i,j+1) \quad . \quad (8) \end{aligned}$$

In Eq. (8), parameters $e_p(i,j)$ and $e_r(i,j)$ are eliminated, and the height $F(i+1,j+1)$ of a point $(i+1,j+1)$, is calculated from known heights $F(i,j)$, $F(i,j+1)$, $F(i+1,j)$ of three points along with sensor outputs $m_A$–$m_D$. Therefore, 2-D profile of the surface being tested can be reconstructed without monitoring pitching and rolling motions, except during the first scan. Consequently, reference mirror bars are not required, and small mirrors could, instead, be used; this is a practical merit of the SLFP method when used in on-machine measurements.

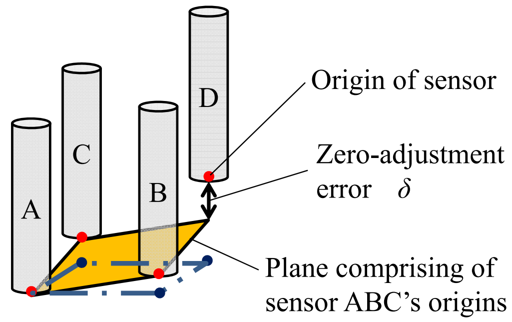



**Fig. 3.** Zero adjustment error in four-point method.

## 3. Zero-Adjustment Error and Self Calibration

### 3.1. Zero-Adjustment Error of SLFP Method

In the SLFP method, zero-adjustment error $\delta$ can be defined as the relative height of the origin of sensor D with respect to the plane comprising origins of sensors A, B, and C, as depicted in **Fig. 3**.

This error in relative height accumulates in proportion to the number of data connections performed using the recurrence formula (Eq. (8)).

### 3.2. Numerical Simulation of the SLFP Method

To confirm the influence of the zero-adjustment and motion errors, numerical simulations were performed using the MATLAB package. Following simulation conditions were used. The translational error was considered as a normal distribution with a standard deviation of 1 $\mu$m. Other normally distributed translational errors, standard deviations of which equaled 0.5 $\mu$m, were considered as errors caused during pitching and rolling motions. The zero-adjustment error measuring 1 $\mu$m was also given. **Fig. 4** depicts the profile of a virtual sample while **Fig. 5** demonstrates an example of sensor output.

**Figure 6** depicts profiles reconstructed based on simulated sensor outputs. **Fig. 6(a)** depicts the case in the absence of the zero-adjustment error. The sample profile was perfectly reconstructed without the influence of translational and posture errors. **Fig. 6(b)** depicts a reconstructed sample profile in the presence of a zero-adjustment error measuring 1 $\mu$m. As observed, the accumulated value of the zero-adjustment error equaled 100 $\mu$m, and the sample shape was buried. In view of this result, it was confirmed that compensation for the zero-adjustment error is extremely important when using the proposed SLFP method.

### 3.3. Self Calibration of Zero-Adjustment Error

The authors propose use of a self-calibration method to determine the zero-adjustment error. This method makes use of the difference between accumulated errors of the two point method and the SLFP method.

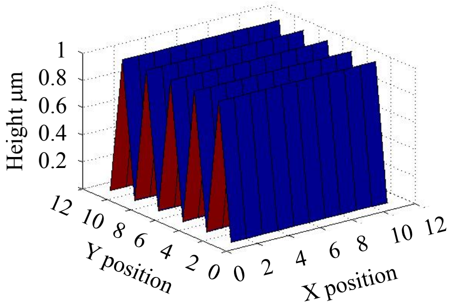



**Fig. 4.** Profile of virtual sample.

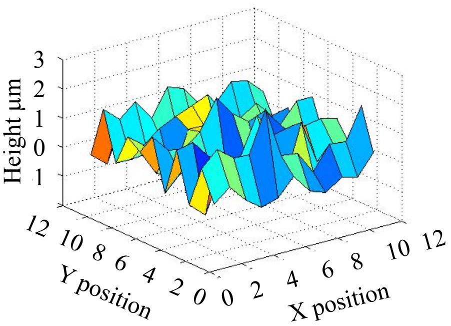
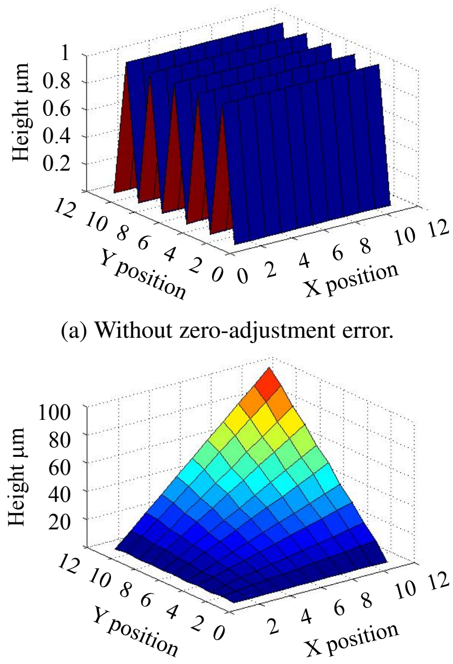



**Fig. 5.** Example of sensor output.




(a) Without zero-adjustment error.

(b) With zero-adjustment error.

**Fig. 6.** Profiles reconstructed via SLFP simulations.

In this method, a square measurement region containing the point $(1,1)$, initial position of a $XY$ scanning, is selected for investigation.

As a first step, the height $F_F(n,n)$ of the end point $(n,n)$ is calculated using the SLFP method, and the obtained 2-D profile contained a twisted component. The error incurred when using the SLFP method attains a maximum value of $(n-1)^2\delta$ at the end measuring point $(n,n)$; i.e.,

$$F_F(n,n) = R(n,n) + (n-1)^2\delta \quad \text{(9)}$$

where $R(n,n)$ denotes the physical height of the endpoint $(n,n)$.

In the second step, the height of the end point can be calculated – using the same dataset being acquired in the SLFP method – by employing the two-point method along the diagonal line connecting the start $(1,1)$ and end $(n,n)$ points. Using this method, height $F_T(n,n)$ of the end point can be expressed as follows.

$$F_T(n,n) = R(n,n) + (n-1)\delta \quad \text{(10)}$$

Equations (9) and (10), both, provide an estimate of the calculated height of the end point $(n,n)$, any difference between heights predicted by Eqs. (9) and (10) could be attributed to the zero-adjustment error and difference between corresponding accumulated errors. The zero-adjustment error $\delta$ can, therefore, be calculated backwards using Eq. (11).

$$\delta = \frac{F_F(n,n) - F_T(n,n)}{(n-1)(n-2)} \quad \text{(11)}$$

By compensating for the zero-adjustment error, 2-D profile of the surface being tested can be appropriately measured.

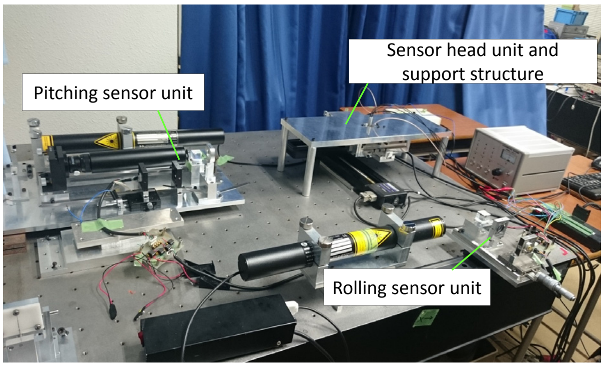



**Fig. 7.** Experimental system.

## 4. Experiment

### 4.1. Experimental System

**Figure 7** depicts a picture of the experimental system used in this study for implementing the SLFP method and zero-adjustment compensation while **Fig. 8** depicts a close up of the region surrounding the sensor head. The experimental system comprised four capacitance-type displacement sensors placed at the vertices of a square, with sides measuring 7 mm, thereby constituting the sensor head installed on a bridge-shaped support structure.

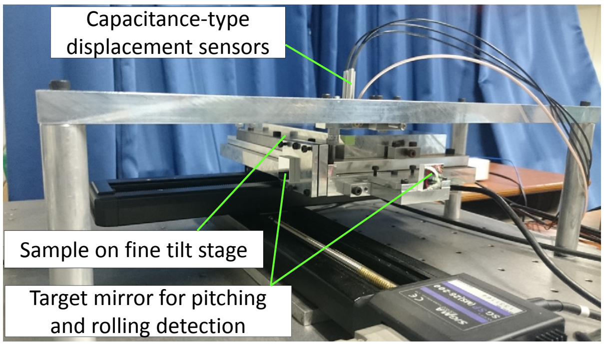



**Fig. 8.** Close up of region surrounding sensor head.

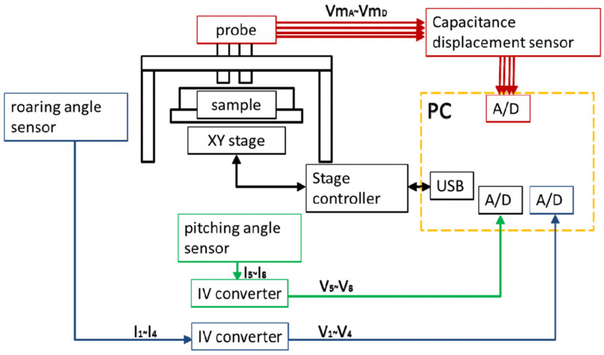



**Fig. 10.** System diagram.

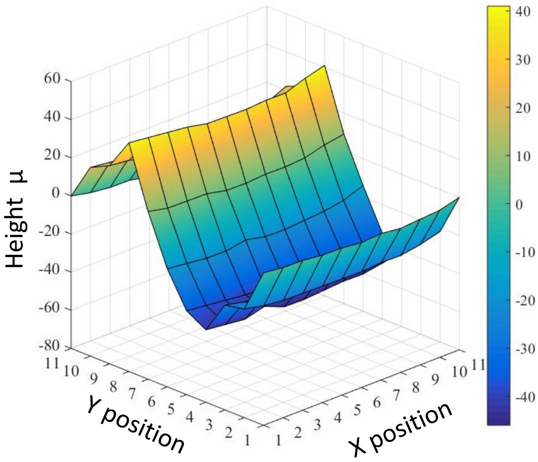



**Fig. 9.** Profile of the target measurement area acquired by a CMM.

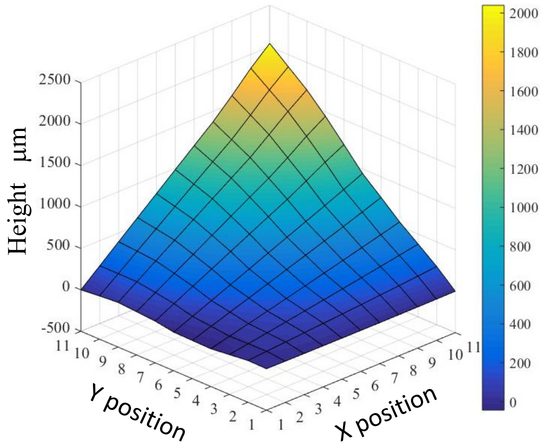



**Fig. 11.** Reconstructed profile without compensation for zero-adjustment error.

The sample (120 × 220 mm) to be measured was mounted on the fine-tilt stage and fixed onto a table of stacked motorized linear stages: a $X$-stage and a $Y$-stage. **Fig. 9** depicts the profile of the target measurement area acquired using a commercially available coordinate measuring machine (CMM) to serves as reference.

Two autocollimators were installed to monitor pitching and rolling motions of the stage table. It must be noted that target mirrors of both autocollimators were small, and use of large bar mirrors was not required because they would only be used for monitoring pitching motion during the first scan along the $X$-line and corresponding rolling motion during $Y$-line scanning.

**Figure 10** depicts a system diagram of the experimental system. The sequential scanning motion with a 7-mm pitch equal to the interval of sensors was controlled by means of a personal computer (PC) through a stage controller.

Outputs from the four displacement and two angle sensors were acquired by the PC by means of a 16-bit analog-to-digital converter board.

### 4.2. Experimental Result

**Figure 11** depicts results obtained without compensation for the zero-adjustment error, wherein the height

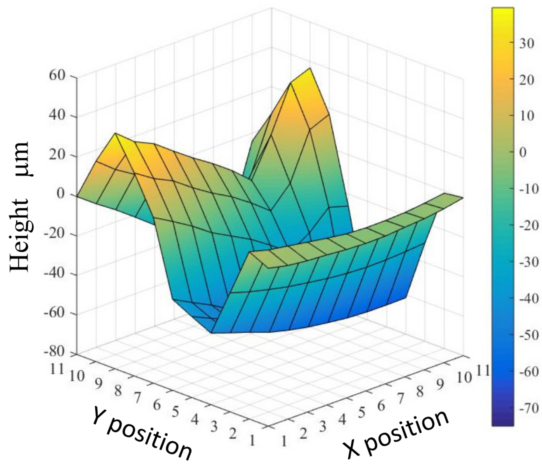



**Fig. 12.** Reconstructed profile with compensation for zero-adjustment error.

of the endpoint was observed to measure approximately 2500 $\mu$m. The sinusoidal shape of the measured surface was not detected. In contrast, the said sinusoidal shape was successfully reconstructed in **Fig. 12**, which depicts results calculated using the proposed calibration method for the zero-adjustment error, thereby demonstrating its validity. In **Fig. 12**, some distortion of the reconstructed profile was observed when compared to **Fig. 9**. The cause of this distortion is thought to be the effects of linearity errors and output drifts of the sensors.

## 5. Conclusions

This study proposes a novel on-machine measurement method – called the square-layout four-point (SLFP) method with angle compensation – for evaluating 2-D profiles of flat machined surfaces. The said method is based on the two-point method with angle compensation and facilitates connection of straightness-measurement results obtained for each scanning line through use of two additional sensors along with an angle sensor. Results of numerical simulations demonstrate that accumulation of the zero-adjustment error incurred by displacement sensors results in unignorable distortion of measured profile shapes. To eliminate this limitation, a new method for self-calibration of the zero-adjustment error has also been proposed, the validity of which has been confirmed via simulations as well as experiments.

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