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Paper:

Evaluation of Pitch Deviations with Comprehensive Representation Suitable for Engagement Evaluation in Different Types of Gears

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Gear tooth flank deviations should be characterized to evaluate individual gear accuracy directly linked to gear performance during engagement. The comprehensive pitch deviation representation we propose is calculated using multiple tooth flank surface deviations as 0-order form deviations. In this representation, pitch error is expressed by angle unit, not by length unit, and calculated from measured conventional deviation (profiles and leads) without measuring pitch deviation. For spur and helical gears, pitch deviation is expressed by a single length unit and also by a single angle unit. On the other hand, for bevel gear flank, pitch deviation expressed by length unit consists of many different values even on a single flank. Using the angle unit expression, form deviation is described exactly by a single parameter. The comprehensive representation we propose overcomes the disadvantages of conventional pitch deviation evaluation, going right to the point of gear engagement evaluation.

Keywords: gear metrology, pitch deviation, comprehensive representation, engagement evaluation

1. Introduction

The key to reducing gear-component noise emission, especially in the automobile industry, lies in gear vibration of gear boxes caused by tooth flank deviation, including tooth deflection and manufacturing error. We studied tooth flank deviation to derive meaningful parameters for evaluating individual gear accuracy directly related to gear performance during engagement. The gear transmits power, torque and rotation highly efficiently. Since it has less energy loss, it is important in saving energy. Being able to evaluate tooth flank topography with simple parameters and predict gear performance using these parameters is useful both in gear design and in feedback to manufacturing processes. The pitch deviation representation we propose is the lowest order form deviation. Pitch deviation is conventionally measured separately from tooth flank topography [1] at a single flank point, at or near the pitch point and expressed by length unit [1–3]. Using only a single gear flank point, however, causes high uncertainty due to surface-irregularity influence.

We begin by outlining how comprehensive pitch deviation is derived [4], then concretely evaluate the new pitch deviation representation. We then discuss the advantages of our proposed representation in gear engagement evaluation.

2. Comprehensive Pitch Deviation Representation

Evaluating pitch deviation based on a single gear flank point presents problems due to dust, lint, scratches, or asperities at the point measured, directly adding the influence of these irregularities to pitch deviation. Since evaluating gear meshing by such a single point is inadequate and useless, this is avoided by using multiple deviations measured at different flank points.

This has the advantage of not having to measure pitch deviation separately, but calculating from measured conventional deviation automatically, where it is considered a lower order form deviation. Here we call pitch deviation described by length unit a 0-order form deviation.

Given the idea of new pitch deviation representation, parameters for expressing pitch deviation are derived using multiple conventional measured deviations on a tooth flank. This is calculated using the least squares method by minimizing the sum of squares of measured deviation. This corresponds to finding the best possible rotational tooth flank location to fit nominal data points – discrete points on a designed tooth flank considered to be correct in manufacture. The objective function of the least squares method is as follows:

$$J(\theta) = \sum_{i} \delta_{i,\mathrm{new}}^2 \qquad (1)$$

$\delta_{i,\mathrm{new}}$ is a new deviation in the direction normal to the tooth flank at point i calculated from nominal data coordinates at lattice point i and those of the measured flank surface at the same lattice point i rotating it by θ. Angle $\theta_{\min}$, which minimizes this objective function, becomes the parameter for 0-order form deviation.

Since rotational angle θ is very small, $\delta_{i,\text{new}}$ is expressed as the orthogonal projection of direction vector of rotation $\mathbf{t}_i$ onto normal vector $\mathbf{n}_i$ of a tooth flank at lattice point i as follows:

$$\delta_{i,\text{new}} \cong \delta_\theta + \delta_i = \mathbf{t}_i \cdot \mathbf{n}_i + \delta_i \quad \dots\dots \quad (2)$$

δ_i is measured original deviation at lattice point i, δ_θ a small deviation along the direction normal to the tooth flank caused by rotating coordinate points of the measured original flank surface at lattice point i, $\mathbf{n}_i$ a unit normal vector at lattice point i, and $\mathbf{t}_i$ a direction vector of rotation at lattice point i around the gear's rotational axis (Z-axis).

When a perpendicular is dropped from lattice point i to the Z-axis, perpendicular vector $\mathbf{r}_i$ is written based on coordinates $(x_i,\ y_i,\ z_i)^T$ of lattice point i as follows:

$$\mathbf{r}_i = (x_i,\ y_i,\ 0)^T \quad \dots\dots\dots\dots \quad (3)$$

The direction of vector of rotation $\mathbf{t}_i$ is toward the tangent at lattice point i of a circle, which has radius $|\mathbf{r}_i|$, so $\mathbf{t}_i$ is always perpendicular to $\mathbf{r}_i$. It is also in a plane whose normal is on the Z-axis. The length of $\mathbf{t}_i$ equals $|\mathbf{r}_i|\,\theta$, yielding

$$\mathbf{t}_i = \theta \begin{pmatrix} -y_i \\ x_i \\ 0 \end{pmatrix} = \theta\, \tau_i \quad \dots\dots\dots \quad (4)$$

Eq. (2) is rewritten as follows:

$$\delta_{i,\text{new}} \cong \delta_\theta + \delta_i = \theta\, \tau_i \cdot \mathbf{n}_i + \delta_i \quad \dots\dots \quad (5)$$

Since inner product $\tau_i \cdot \mathbf{n}_i$ of two vectors is scalar, meaning independent of θ, we put it as g_i. A new deviation is simply written as follows:

$$\delta_{i,\text{new}} \cong g_i\theta + \delta_i \quad \dots\dots\dots\dots \quad (6)$$

Scalar $g_i = \tau_i \cdot \mathbf{n}_i$ is calculated directly by using given nominal data. Since δ_i is measured original deviation, obtained independent of θ, Eq. (6) is a function only for θ. Objective function $J(\theta)$ becomes a 2nd degree polynomial for θ, so the angle minimizing the objective function is obtained analytically as follows:

$$\theta_{\min} = -\frac{\sum_i \delta_i g_i}{\sum_i g_i^2} \quad \dots\dots\dots\dots \quad (7)$$

$\theta_{\min}$ is the 0-order form deviation expressed by angle unit and $g_i\theta_{\min}$ expressed by length unit as deviation.

For spiral bevel and hypoid gears, since g_i differs for each lattice point on a flank such as a freeform surface, 0-order form deviations are, generally and strictly speaking, not real offset values on the deviation plane. For spur and helical gears, which have involute helicoidal tooth flanks, the value of orthogonal projection g_i is always the same on a flank. Let f_{pt} be single pitch deviation at arbitrary radius r_x, α_x be a pressure angle at that radius, and r_b be a radius of a base circle; then

$$\delta_\theta = f_{pt} \cdot \cos\alpha_x = r_x\theta \cdot \cos\alpha_x = r_b\theta \quad \dots \quad (8)$$

Zero-order form deviations expressed by length unit are constant at all lattice points on the plane of action, and, are real offset values. That is,

$$g_i = r_b = \text{const.} \quad \dots\dots\dots\dots \quad (9)$$

Given this consideration, when the number of total measurement points on a tooth flank is represented by n, Eq. (7) is expressed as follows:

$$\theta_{\min,\,\text{inv}} = -\frac{\sum_i \delta_i r_b}{\sum_i r_b^2} = -\frac{1}{nr_b}\sum_i \delta_i \quad \dots\dots \quad (10)$$

The 0-order form deviation expressed by an angle unit is expressed as a simple arithmetic average of measured original deviations δ_i divided by the radius of a base circle.

Using angle representation, 0-order form deviations are described by a single parameter precisely and comprehensively, so it is best to express 0-order form deviations (pitch error) using an angle unit rather than a length unit. Since multiple measured deviations are part of the calculation of the pitch deviations, our proposed pitch deviation representation eliminates conventional pitch deviation measurement faults, and all flanks of an individual gear usually have similar topography based on its manufacturing process. This new definition also goes right to the point of gear meshing and manufacturing mechanisms.

3. Example of Pitch Representation

Actual gear flanks are measured with the Carl Zeiss IMT Corp.'s Prismo Vast CMM [5]. The gear measured is a spiral bevel gear with specifications $m_n = 6.5140$, $z = 39$, $\beta = 39.829°$, and $b = 58.000$ mm, manufactured using Oerlikon cutting. Nominal data consists of 11 $\times$ 11 lattice points for each flank. Left flanks are measured for four teeth every 90° for individual gears.

Measured tooth flank deviations are shown in **Fig. 1**. Deviations project from the tooth flank onto the deviation plane, where nominal tooth flank points are represented as equally spaced rectangular lattices. The right side along the tooth trace corresponds to the toe, and the left to the heel. Calculated 0-order form deviations are shown in **Fig. 2**. Calculated pitch deviation expressed by angle unit is $\theta_{\min,S1} = -32.3'' = -156\ \mu$rad.

In the next example, measured deviations from a different tooth flank on the same gear are shown in **Fig. 3** and extracted pitch deviations in **Fig. 4**. These deviations are smaller than in **Fig. 2**. Calculated pitch deviation in **Fig. 4** expressed by angle unit is $\theta_{\min,S2} = -15.0'' = -72.7\ \mu$rad.

The two calculated pitch deviations yield partial cumulative pitch deviation θ_{Fp} expressed by angle unit based on the tooth flank in **Fig. 1**. $\theta_{Fp} = \theta_{\min,S2} - \theta_{\min,S1} = 17.3'' = 83.3\ \mu$rad.

Conducting the same calculation against mutually adjacent teeth yields single pitch deviation.

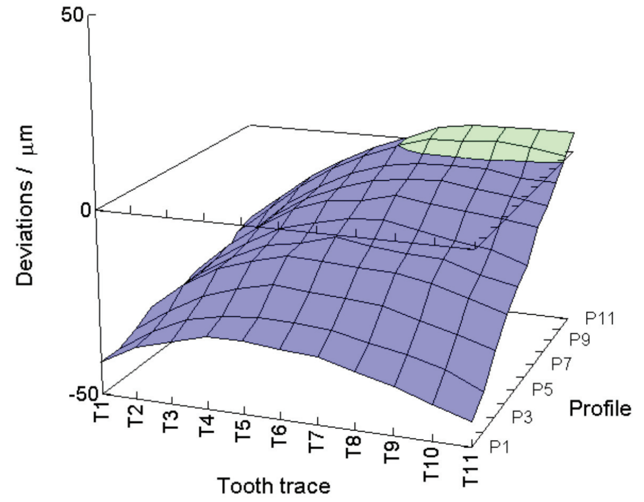


Fig. 1. Measured bevel gear tooth surface deviations.

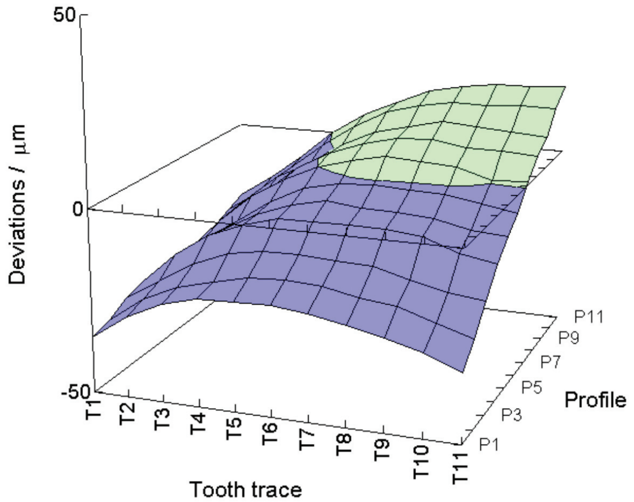


Fig. 3. Measured deviations for a different tooth flank.

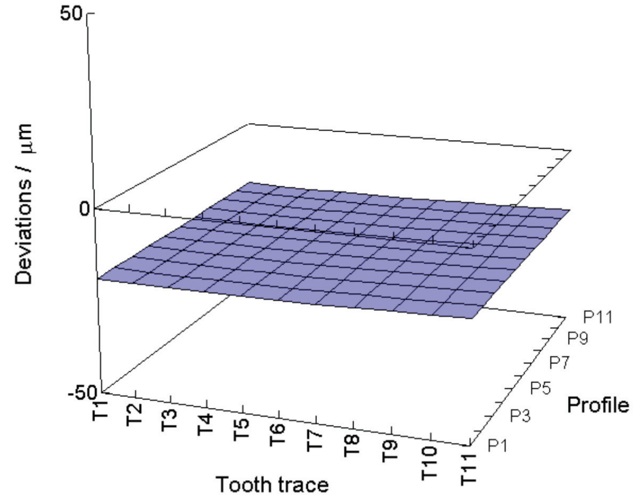


Fig. 2. Calculated form pitch deviation from **Fig. 1**.

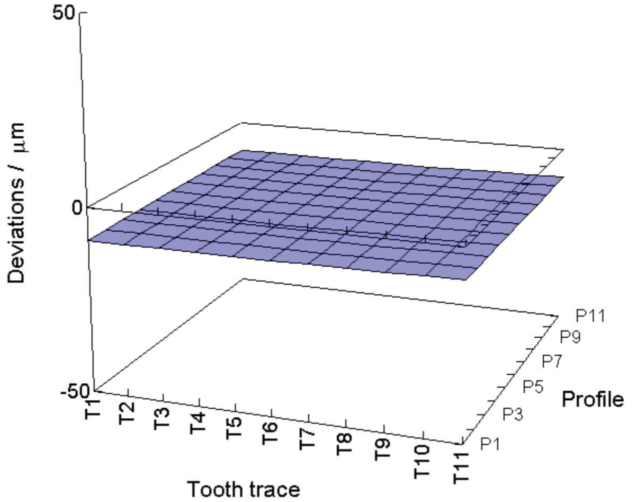


Fig. 4. Calculated form pitch deviation from **Fig. 3**.

4. Discussion

Our proposed angle representation has advantages in characterizing tooth flank topography and evaluating gear engagement. Considering gear pair meshing mechanisms, transmission error becomes highly useful information. Transmission error is defined as the transmitted rotational angle difference between a gear pair's input and the output shafts. It is sometimes expressed by angle unit and by length unit, so pitch deviation expressed by angle unit is easily compared directly to transmission error.

Pitch deviation tolerance may be unified and treated as the same value for different module gears using the angle unit as a size independent tolerance. This is applicable to all types of gears having arbitrary tooth flank forms.

In characterizing tooth flank topography, the obtained pitch deviation is subtracted from original measured deviations and resulting deviations are regarded as higher order form deviations eliminated by the influence of pitch deviation. Subtracted resulting deviations between **Fig. 1** and **Fig. 2** are shown in **Fig. 5** and those between **Fig. 3** and **Fig. 4** in **Fig. 6** shown similarly.

Comparing **Figs. 5** and **6** shows the resemblance of resulting deviations. In gear manufacture, cutting edges on a gear cutter or grains on a grinding wheel run on predefined paths to produce the intended gear teeth. Any misaligned machine settings or machine tool movement error adverse affect the shapes of tooth flanks. This typical example indicates that trends in error influence are similar on every tooth flank. A slight difference is also seen between **Figs. 5** and **6**, possibly due to cutting edges chipping, inadequate cutting edges regrinding, or inadequate grinding wheel dressings. Such evidence appears as cutting feed marks, waviness, roughness, or cutting texture.

In the above example, $\theta_{\min}$ is calculated using all lattice points on the tooth flank in **Fig. 7 (a)**. It is also possible, however, to select some specified points partially to evaluate pitch deviation in a contact area as shown in **Fig. 7 (b)**, for example.

Even here, calculating pitch deviation is the same, with

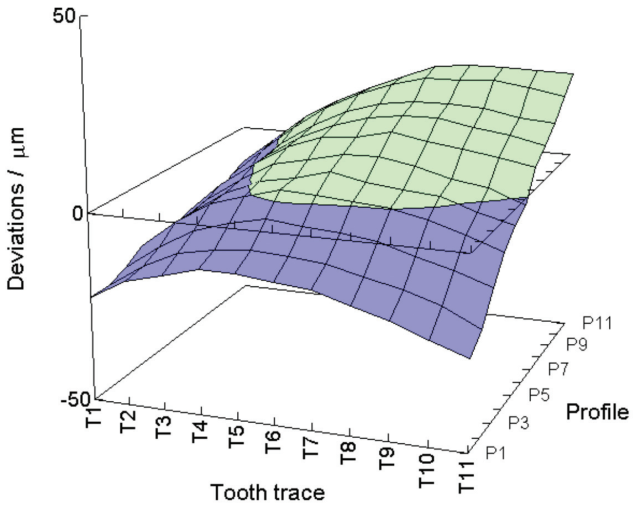


Fig. 5. Resulting higher order deviations for **Fig. 1**.

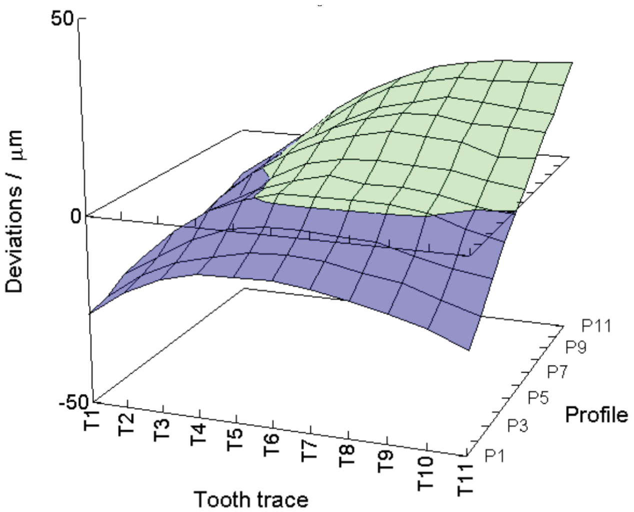


Fig. 6. Resulting higher order deviations for **Fig. 3**.

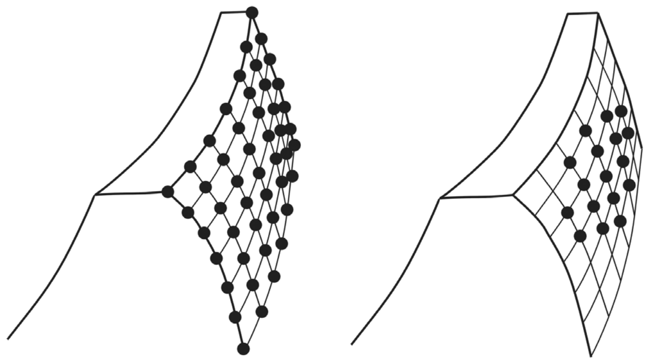
only the number of measured points considered in calculation differing. Using the weighting function for each lattice point is also applicable, if the significant level of each point is not the same in gear engagement.

5. Conclusions

To overcome the disadvantage of conventional pitch measurement and evaluation of pitch deviations, we propose comprehensive pitch deviation representation in which pitch deviation is calculated with multiple measured surface deviations on tooth flanks. Actual bevel gear flanks are measured and pitch deviations proposed are calculated, confirming the feasibility of the parameter.

In short:

1. Our proposed pitch deviation representation is described by a single parameter using an angle unit. It is applicable to unify the pitch deviation evaluation



(a) All points (b) Some points

Fig. 7. Variety of selected points to calculate pitch deviation.

for different module gears.

2. This representation has the advantage of eliminating the need to measure pitch deviation separately because pitch deviation is calculated from measured conventional surface deviations automatically.
3. Large uncertainty dependent on a single measured value is avoided by using multiple surface deviations measured at different points on a tooth flank.
4. Specified points can be selected to evaluate pitch deviation and calculation of the pitch deviation is exactly the same.

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