

Paper:

HOSVD Based Canonical Form for Polytopic Models of Dynamic Systems

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[Received June 6, 2007; accepted February 18, 2008]

The higher order singular-value-based canonical form of linear parameter varying models we define, extracts a models's most important invariant characteristics. We studied the numerical reconstructibility of the canonical form using a routinely executable tractable uniform method, and present convergence theorems for given numerical reconstruction constraint.

Keywords: LPV model, polytopic dynamic model, TP model, HOSVD, linear matrix inequalities (LMI)

1. Introduction

Modern control theory focuses in part on analysis and control design based on linear matrix inequalities (LMI) in linear parameter varying (LPV) system representation. The currently most widespread approach is to seek robust, efficient nonlinear control results, e.g. [1, 2, 5]. These approaches usually prepare the system model for LMI design based on analytical derivation, which requires a series of idiosyncratic, rather sophisticated analysis. Although we have analytical solutions, the final LMI design result and controller implementation based on numerical calculation is, in the final realization, approximate.

In contrast, the recently proposed uniformly and automatically executable tensor product (TP) model transformation prepares a given model in the polytopic forms to which LMI design is immediately applicable. The mathematical structure of our proposal method is based on the recently developed higher order singular value decomposition (HOSVD) concept.

After introducing a canonical form of LPV models that extracts important invariant system characteristics, we discuss this canonical form as a new nonlinear control theory concept, i.e., an invariant representation of LPV models that provides uniform, tractable, readily executable numerical ways and creative manipulations to generate convex (polytopic) representations of LPV models. This may help to achieve better observability and controllability, and an important objective is to develop reliable, numerically appealing algorithms to reconstruct this canonical

form.

Our main objective is thus to prove that the canonical form is numerically reconstructible using TP model transformation. We present convergence theorems based on different numerical constraints and settings of numerical reconstruction.

Note that the canonical form and the method of its numerical reconstruction could mediate between important analytical models and widely used heuristically identified models.

2. Background

The higher order singular value decomposition (HOSVD) based canonical form of linear parameter varying (LPV) dynamic models and the tensor product (TP) model transformation was developed due to significant paradigm changes in control theory, mathematics, system modeling, and identification theories appearing almost simultaneously in the last few decades.

Tensor decomposition: During the last 150 years, several mathematicians, including Beltrami, Jordan, Sylvester, Schmidt, and Weyl have been responsible for establishing the foundations of singular value decomposition (SVD) and for developing SVD theory – one of the most fruitful developments in linear algebra. A very recent result is the higher-order generation of the SVD (HOSVD) form of higher-order tensors (Lathauwer, 2000, [6]). The Workshop on Tensor Decompositions and Applications held in Luminy, Marseille, France (2005), was the first event in which HOSVD was the key topic. Its very unique power comes from the fact (1) that it can decompose a given N-dimensional tensor into a full orthonormal system based on a special ordering of higher-order singular values, and (2) it expresses the rank properties of the tensor in the order of its \mathcal{L}_2 -norm. In effect, HOSVD, instead of canonically decomposing tensors, extracts the very clear, unique structure underlying a given tensor.

TP model transformation: We conclude that we have, on the one hand, powerful optimization and control de-

sign for polytopic and affine forms of LPV models, and, on the other, a large variety of identification techniques. These two aspects can hardly be linked, however, due to the lack of uniform representation. A need thus exists for automatic, uniform ways to convert alternative representations to unique polytopic or affine forms.

TP model transformation, proposed as a possible solution [3, 4], transforms given LPV models into proper polytopic forms upon which LMI-based design techniques are immediately executable. TP model transformation yields a TP model that belongs to the class of polytopic models in which parameter-dependent weighting of the vertex systems involves one-dimensional functions of parameter vector elements. This provides a relatively simple way to describe convex hull generations in terms of matrix operations, and is widely used in B-spline based control PDC, and further fuzzy control design [9].

TP model transformation has the following properties:
2.1 It is executed uniformly, regardless of whether the model is given in the form of analytical equations resulting from physical considerations or as an outcome of soft-computing-based identification such as neural networks or fuzzy-logic-based methods or as a result of black-box identification without analytical interaction, within a reasonable time.

2.2 It generates the HOSVD-based canonical form of LPV models [8] – a unique representation **that extracts the unique structure of a given LPV model – in the same sense as the HOSVD does for tensors and matrices – so that:** i) the number of LTI components is minimized; ii) weighting functions are univariate functions of the parameter vector in an orthonormed system for each parameter; iii) LTI systems are in orthogonal position; and iv) LTI systems and weighting functions are ordered based on higher-order singular values of the parameter vector.

2.3 TP model transformation has been extended to generate different types of convex polytopic models [4] because the type of convexity of polytopic models greatly influences LMI-based design feasibility and resulting performance. This means that instead of developing new LMI equations for feasible controller design – a widely used approach – we can focus on systematic numerical and automatic modification of the convex hull. Note that both TP model transformation and LMI-based control design are sequentially numerically executable, enabling a broad class of problems to be solved in a straightforward, tractable, numerical way.

2.4 TP model transformation is executed before using LMI design, meaning that when we start LMI design, we already have global weighting functions and, during control, we need not determine local weighting of LTI systems for feedback gains to calculate the control value at every point of hyperspace the system goes through. Pre-defining continuous weighting functions also ensures that no friction occurs in weighting during control.

3. Finite Element TP model

Consider the following linear parameter-varying state-space model:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \mathbf{S}(\mathbf{p}(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix} \dots \dots \dots (1)$$

with input $\mathbf{u}(t)$, output $\mathbf{y}(t)$, and state vector $\mathbf{x}(t)$. System matrix $\mathbf{S}(\mathbf{p}(t)) \in \mathbb{R}^{I_{N+1} \times I_{N+2}}$ ($I_{N+1} = O$, $I_{N+2} = I$) is a parameter-varying object, where $\mathbf{p}(t) \in \Omega$ is a time varying N -dimensional parameter vector and is an element of closed hypercube $\Omega = [a_1, b_1] \times [a_2, b_2] \times \dots \times [a_N, b_N] \subset \mathbb{R}^N$. Function $\mathbf{p}(t)$ can also include elements of state-vector $\mathbf{x}(t)$, so (1) is considered in the class of nonlinear dynamic state-space models.

Definition 1: finite element TP model: $\mathbf{S}(\mathbf{p}(t))$ of (1) is given for any parameter $\mathbf{p}(t)$ as the convex combination of LTI system matrices. \mathbf{S} is also called *vertex systems*:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = (\mathbf{S} \boxtimes_{n=1}^N \mathbf{w}_n^T(p_n(t))) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix} \dots \dots \dots (2)$$

where column vector $\mathbf{w}_n(p_n) \in \mathbb{R}^{I_n}$, $n = 1, \dots, N$ contains one variable bounded and continuous weighting functions $w_{n,i_n}(p_n)$, ($i_n = 1, \dots, I_n$). Weighting function $w_{n,i_n}(p_n(t))$ is the i_n -th weighting function defined on the n -th dimension of Ω , and $p_n(t)$ is the n -th element of vector $\mathbf{p}(t) = (p_1(t), \dots, p_N(t))$. $I_n < \infty$ denotes the number of weighting functions used in the n -th dimension of Ω . The dimensions of Ω are assigned to elements of parameter vector $\mathbf{p}(t)$. $(N + 2)$ -dimensional coefficient (system) tensor $\mathbf{S} \in \mathbb{R}^{I_1 \times \dots \times I_{N+2}}$ is constructed from LTI vertex systems

$$\begin{aligned} \mathbf{S}_{i_1 \dots i_N} &= \{S_{i_1 \dots i_N, \alpha, \beta}, 1 \leq \alpha \leq I_{N+1}, 1 \leq \beta \leq I_{N+2}\} \\ \mathbf{S}_{i_1 \dots i_N} &\in \mathbb{R}^{I_{N+1} \times I_{N+2}} \end{aligned}$$

For details, see [3, 4]. Note that we use \boxtimes_n instead of symbol \times_n given in [6] for the notation of the n -mode tensor-matrix product. The new notation introduced has no relationship to other similar symbols, e.g., the Kronecker product.

4. HOSVD-Based Canonical Form of Finite Element TP Models

Consider LPV model (1), which can be given in the finite element TP model form (2), i.e., matrix valued function $\mathbf{S}(\mathbf{p})$ can be given as:

$$\mathbf{S}(\mathbf{p}) = \mathbf{S} \boxtimes_{n=1}^N \mathbf{w}_n^T(p_n),$$

where $\mathbf{p} = (p_1, \dots, p_N) \in \Omega$.

For this model, we assume that functions $w_{n,i_n}(p_n), i_n = 1, \dots, I_n, n = 1, \dots, N$, are linearly independent (in the means of $\mathcal{L}_2[a_n, b_n]$) over the intervals $[a_n, b_n]$. In the opposite case, we choose linearly independent functions from $w_{n,i_n}(p_n), i_n = 1, \dots, I_n$ and express remaining functions with their help in linear form. This means that the original TP model can also be given with linearly independent functions.