

Paper:

Further Results on T-S Fuzzy Controller Design Subject to Input Constraint

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In general, when using the Takagi-Sugeno (T-S) fuzzy model to develop a control system, the state feedback control gain can be obtained by solving some linear matrix inequalities (LMIs). In this paper, we consider a class of nonlinear systems with input constraint (saturation). To obtain the control gain, we require to employ certain extra LMIs besides the general ones. As a result, all the LMIs are more conservative. At the same time, one of the extra LMIs confines the initial state to a region, which is referred to as an ellipsoid, and is relevant to a matrix variable in the LMIs. Therefore, the goals of this paper are: 1) making the ellipsoid as large as possible so that the initial state can be confined to the region easily and; 2) making all the LMIs more feasible to obtain the control gain.

Keywords: fuzzy controller, input constraint, ellipsoid, LMIs conservatism

1. Introduction

One of the essential elements of any control problem is the model of the dynamical system to be controlled. In many cases a mathematical model of the system is unavailable or incomplete, or the equations that we believe are adequate, to represent the behavior of the system are too complicated for the design purpose. This is the most likely reason why the application of the fuzzy set theory to control problems has been the focus of numerous studies. The reason is that the fuzzy set theory provides an alternative approach to the traditional modeling and design of control systems, where knowledge at the dynamic model of the system in the traditional sense is uncertain and time-varying. In recent years, there have been significant advances in the study of the stability analysis and controller synthesis for the so-called Takagi-Sugeno (T-S) fuzzy systems, also known as the Takagi-Sugeno-Kang (TSK) fuzzy systems [1, 2], which have been used to represent certain complex nonlinear systems. In the T-S fuzzy model, the local dynamics in different state-space regions are represented by linear models. The overall model of the system is obtained by the fuzzy blending of these local models. The control design is carried out

based on the fuzzy model by the so-called parallel distributed compensation (PDC) scheme [3]. For each local linear model, a linear feedback control is designed. The resulting overall controller is again a fuzzy blending of the individual linear controllers. Originally, Tanaka and his colleagues have provided certain conditions that are sufficient for the stability of the T-S fuzzy systems in the sense of Lyapunov [3–6]. The conditions for the existence of a common Lyapunov function are obtained by solving the linear matrix inequalities (LMIs).

In this paper, we consider a class of nonlinear systems with input constraint (saturation). In other words, the norm of the input vector to the system is less than or equal to a prescribed limit. Actually, the system input saturation can severely degrade the closed-loop system performance and sometimes, even destabilizes the otherwise stable closed-loop system. Recently, considerable attention has been paid to the systems with input saturation [12–14]. In general, input saturation problem can be overcome by either designing low gain control laws, which require that the saturation limits be avoided for a given bound on the initial conditions [5], or by estimating the domain of attraction in the presence of input saturation [14]. In this paper, we focus our attention on the bounded initial conditions, and make an effort to ease them.

On the other hand, the conditions for the existence of a common Lyapunov function for all the local models in the LMIs approach sometimes tends to be very conservative; this is because the common Lyapunov function may not exist at all for many systems, particularly for those used to represent highly nonlinear complex systems. Furthermore, in this paper, in order to deal with the input constraint, some extra LMIs are required; it is therefore more necessary to relax the LMI conditions. To relax the LMIs conditions, Tanaka, Ikeda and Wang [6] utilized a fuzzy system's property Eq. (3) and reported a pioneering work in this area. Kim and Lee [7] considered the interactions among the subsystems and addressed them in a single matrix; as a result, the conservatism due to the interactions is reduced. Subsequently, Teixeira, Assuncao, and Avellar [8] reported an extended version of Kim and Lee's method. To the best of our knowledge, the method proposed by Teixeira et al. is the most effective for relaxing the LMI conditions.

The rest of this paper is organized as follows. In section 2, the T-S fuzzy model and a basic theorem regarding the involvement of the T-S fuzzy model into control design are presented, and the problem statement considered in this paper is described. In section 3, after leading the control design to the LMI approach, attention is paid to 1) easing the LMI conditions so that the system is quadratically stable in the large; 2) the relaxation of the LMI conditions so that the LMI problem is more feasible. In section 4, a simulation is provided to demonstrate the effectiveness of the proposed design techniques. Finally, section 5 concludes this paper.

2. T-S Fuzzy Model and Problem Statement

The T-S fuzzy model is described by the fuzzy IF-THEN rules, which represent local linear input-output relations of a nonlinear system as follows [1–3].

$$\text{Rule } i: \text{ IF } x_1(t) \text{ is } M_1^i \text{ AND } \dots \text{ AND } x_n(t) \text{ is } M_n^i \\ \text{ THEN } \dot{x}(t) = A_i x(t) + B_i u(t) \quad \dots \quad (1)$$

where $i (= 1, 2, \dots, r)$ represents i th fuzzy rule; $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathcal{R}^n$, the system state; $u(t) = [u_1(t), u_2(t), \dots, u_m(t)]^T \in \mathcal{R}^m$, the controller; the constant matrices A_i and B_i are of appropriate dimensions; and $M_j^i (j = 1, 2, \dots, n)$, the j th fuzzy set of the i th fuzzy rule with the membership function $\mu_j^i(x_j(t))$. Let

$$w_i(t) = \prod_{j=1}^n \mu_j^i(x_j(t)).$$

Then, given a pair $(x(t), u(t))$, the resulting fuzzy system model is inferred as the weighted average of the local models and has the form

$$\dot{x}(t) = \frac{\sum_{i=1}^r w_i(t) (A_i x(t) + B_i u(t))}{\sum_{i=1}^r w_i(t)} \\ = \sum_{i=1}^r \alpha_i(t) (A_i x(t) + B_i u(t)) \quad \dots \quad (2)$$

where for $i = 1, 2, \dots, r$,

$$\alpha_i(t) = \frac{w_i(t)}{\sum_{i=1}^r w_i(t)} \geq 0, \quad \sum_{i=1}^r \alpha_i(t) = 1. \quad \dots \quad (3)$$

Generally, the following PDC controller [3, 4] can be designed for the T-S fuzzy model Eq. (2).

$$\text{Control Rule } i: \\ \text{ IF } x_1(t) \text{ is } M_1^i \text{ AND } \dots \text{ AND } x_n(t) \text{ is } M_n^i \quad \dots \quad (4) \\ \text{ THEN } u(t) = -F_i x(t),$$

where $F_i \in \mathcal{R}^{m \times n} (i = 1, 2, \dots, r)$ is the controller matrix gain to be designed. The overall state feedback control law is finally represented as

$$u(t) = - \sum_{i=1}^r \alpha_i F_i x(t). \quad \dots \quad (5)$$

Then, the closed-loop T-S fuzzy control system comprising Eqs. (2) and (5) is described by

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r \alpha_i(x) \alpha_j(x) (A_i - B_i F_j) x(t) \\ = \sum_{i=1}^r \alpha_i(x)^2 G_{ii} x(t) \\ + 2 \sum_{i=1}^r \sum_{j>i}^r \alpha_i(x) \alpha_j(x) \left(\frac{G_{ij} + G_{ji}}{2} \right) x(t) \\ = \sum_{i=1}^r \alpha_i(x)^2 \Lambda_{ii} x(t) \\ + 2 \sum_{i=1}^r \sum_{j>i}^r \alpha_i(x) \alpha_j(x) \Lambda_{ij} x(t) \quad \dots \quad (6)$$

where $G_{ij} = A_i - B_i F_j$, and $\Lambda_{ij} = \frac{G_{ij} + G_{ji}}{2}$. Clearly, it holds that $G_{ii} = \Lambda_{ii}$, and $\Lambda_{ij} = \Lambda_{ji}$.

For the stability of control system Eq. (6), the following theorem that provides the basic LMI conditions is applied [3].

Theorem 1: The equilibrium of the continuous fuzzy control system Eq. (6) is quadratically stable in the large if there exists a symmetric matrix P such that

$$P > 0 \quad \dots \quad (7)$$

$$\Lambda_{ii}^T P + P \Lambda_{ii} < 0 \quad (i = 1, 2, \dots, r) \quad \dots \quad (8)$$

$$\Lambda_{ij}^T P + P \Lambda_{ij} \leq 0 \quad \dots \quad (9)$$

$$(1 \leq i < j \leq r, \alpha_i(x) \alpha_j(x) \neq 0).$$

It is noted that throughout this paper, the origin $x(t) = 0$ is assumed to be the only equilibrium point of the fuzzy control system.

In this paper, we consider a general nonlinear system of the form

$$\dot{x}(t) = f(x(t), u(t)) \quad \dots \quad (10)$$

where $f \in \mathcal{R}^n$ is a sufficiently smooth nonlinear function in the state $x(t) \in \mathcal{R}^n$ and control input $u(t) \in \mathcal{R}^m$. Also, the following constraint is forced on the control input at all times $t \geq 0$.

$$\|u(t)\| \leq \mu. \quad \dots \quad (11)$$

As a well-known fact, a nonlinear system resembling Eq. (10) can be effectively approximated in the fuzzy rule manner similar to Eq. (1) with some appropriate membership functions for the fuzzy sets in both antecedent and consequent parts of the plant fuzzy rules. Then, the pending task is to design r local linear state feedback laws Eq. (4) such that the origin of the closed-loop system with the input constraint Eq. (11) is quadratically stable in the large. To this end, we must transfer inequality Eq. (11) into an LMI form that eventually joins the other LMIs such as Eqs. (7)-(9) for the system stability. In addition, the LMI corresponding to Eq. (11) leads to a bounded initial state condition. Actually, as mentioned later on, the initial state condition is very conservative. Therefore, we must ease the condition to allow the number of initial