Paper:

Application of Fuzzy Set Theory and DEA Model to Evaluating Production Efficiency for Taipei City Bus Company

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The purpose of this paper is to present a performance evaluation model for forecasting production efficiency for Decision Making Units (DMUs). This model is based on the fuzzy set theory, fuzzy regression, and the DEA model. A stochastic DEA approach has been proposed and used widely to analyze the performance of the uncertain input or output data, but this approach requires large data samples and assumes probability distribution in measurement error terms. The concept of fuzzy numbers was seldom considered, although the stochastic DEA approach can be used for prediction. This paper integrates fuzzy regression and fuzzy DEA as one model. The results of this research show that the model developed in this paper is applicable to evaluate the "reform policy for passenger loading operations" currently undertaken by the Taipei City Bus Company. Based on this study, the integration of fuzzy numbers, fuzzy regression, and the DEA model can be applied to evaluate production efficiency of the city bus company for the short-term future.

Keywords: DEA, efficiency, fuzzy DEA, DEA forecast model, fuzzy regression model

1. Introduction

The Data Envelopment Analysis (DEA) model was developed by Charnes, et al. (1978). The DEA model formulates the ratio between output and input of resources as mathematical programming to measure relative efficiency. The model will not be affected by the unit of input and output, has no pre-set function form, and can deal with qualitative and quantitative factor problems. The DEA model as extended by Banker (1993), Banker and Thrall (1992), Nash and Sterna-Kareat (1996), and Yu, et al. (1996) has already been widely used to measure efficiency of public and private sectors.

Kao, et al. (1993) applied the model to evaluate efficiency in the public sector. Stewart (1996) considered the impact of the decision-makers' attitude, combining DEA with multi-criteria decision-making in order to determine the scope of weights within the DEA model. Kao (1994) investigated sensitivity analysis in the input and output resources of DMUs. Yu, et al. (1996), developed a generalized DEA model, which investigates possible set attributes of the production.

In regard to the transportation market, Chang and Kao (1992) have used the model to evaluate the production efficiency of public and private city buses. Lan, et al. (1997) has applied the DEA model to evaluate the operation efficiency of port containers. Hjalmarsson and Odeck (1996) have applied the DEA model to evaluate the production efficiency of road freight transportation, considering not only the relationship between multi-inputs and multi-outputs of bus transportation, but also the sensitivity analysis of appraisal efficiency when components of the road freight transportation were changed.

On the other hand, a stochastic DEA approach has been proposed and used widely to analyze the performance of uncertain input or output data by Banker (1986), Sengupta (1987), Retzlaff-Roberts and Morey (1993), Cooper, et al.(1998) and others. Sengupta (1987) focused on the stochastic variations of input and output data, incorporating them into the DEA model, while research by Retzlaff-Roberts and Morey (1993) based on Banker's model led to the development of a stochastic allocative DEA model (ADEA). The stochastic model introduced by Banker, Sengupta, Retzlaff-Roberts and Morey, Retzlaff-Roberts and Morey defines "uncertainty" as the measurement error between the actual data and observed data in input or output variables.

A major criticism of DEA models is that they are deterministic for input or output data and do not allow for uncertainty. The DEA model can only be used in conditions where the input and output are crisp numbers. It is notable that although the stochastic model attempts to modify the crisp number assumption and allows stochastic variation in data, the uncertainty of the variable input or output data must have a known probability distribution with mean and covariance. In addition to the large sample data that should be used for the stochastic model, Nagano, et al. (1995) has proposed that the output should be a fuzzy number under uncertain environments in order to overcome the constraints of the DEA model. From Nagano's findings, it is shown that the output variable can be a fuzzy number when the environment is uncertain. This notion offers the interface to integrate fuzzy set theory and DEA model. However, it should be worth thinking concisely how to integrate the fuzzy set and DEA model so the model can be applied to predict efficiency for a specific transportation company. In addition, the predicted model should consider the multiproduct feature, as the output number becomes a fuzzy number.

This paper has therefore, construed the evaluation model for fuzzy efficiency forecasting so as to appraise production efficiency evaluation. The fuzzy DEA model makes use multiple input variables (vehicle, staff and interim input) and two output variables (operated-mileage, passenger loading). The fuzzy multi-product model will be applied to the Taipei City Bus Company for their "reform operation plan" that is under implementation.

The remainder of this paper is organized as followings: Section 2 discusses the concepts of the DEA model and fuzzy set theory; Section 3 investigates the fuzzy DEA model; Section 4 develops the fuzzy regression and fuzzy DEA models; and Section 5 presents a case study. Conclusions are made in the final section.

2. Concepts of the DEA Model and Fuzzy Regression Model

In this section, we investigate the DEA model, the fuzzy number, and the fuzzy regression model.

2.1. DEA Efficiency Evaluation Model

The DEA model, developed by Charnes, et al. (1978), is a mathematical programming model that modifies defects of the Farrell model, which was unable to handle several inputs and outputs. Within the model, it is assumed that there are n Decision-Making Units (DMUs), with m inputs and p outputs, while the efficiency evaluation model of k^{th} DMU can be defined as in Eq.(1).

$$Max\, f_k = \frac{\sum_{r=1}^{p} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}$$

$$s.t.\ \frac{\sum_{r=1}^{p} u_r y_{rs}}{\sum_{i=1}^{m} v_i x_{is}} \le 1, \quad s = 1, 2, \ldots, n;$$

$$u_r \ge \varepsilon \ge 0, r = 1, 2, \ldots, p;$$

$$v_i \ge \varepsilon \ge 0, i = 1, 2, \ldots, m. \qquad \cdots\cdots (1)$$

where x_{is} is the i^{th} input value for s^{th} DMU; y_{rs} is the r^{th} output value for the s^{th} DMU; u_r and are v_i the virtual multiplier of the output and input, respectively; and ε is a very small positive value. It is difficult to obtain the solution from Eq. (1) because Eq. (1) is a nonlinear programming problem. Therefore, Eq. (1) was modified as Eq.(2) by Charnes, et al., resulting in a linear programming problem where a solution can be more easily obtained.

$$Max\ \theta_k = \sum_{r=1}^{p} u_r y_{rk}$$

$$s.t.\ \sum_{r=1}^{p} u_r y_{rs} - \sum_{i=1}^{m} v_i x_{is} \le 0, \quad s = 1, 2, \ldots, n;$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1;$$

$$u_r \ge \varepsilon \ge 0, r = 1, 2, \ldots, p;$$

$$v_i \ge \varepsilon \ge 0, i = 1, 2, \ldots, m. \qquad \cdots\cdots (2)$$

where θ_k is the efficiency value for k^{th} DMU, and θ_k is a crisp number while x_{ik} and y_{rk} are both crisp number for the DMU. Eq. (1) points out that the input or output number must be crisp number. Thus the DEA model can only deal with the determining decision-making environment for DMU. If input variable x_{is} or output variable y_{rs} is not crisper number, suppose y_{rk} is a fuzzy number, and denote $\tilde{y}_{rk}$. Then the efficiency value θ_k will become a fuzzy solution, denoted the $\tilde{\theta}_k$, and the efficiency value of Eq. (2) will become $\tilde{\theta}_k = \left(\theta_k^L, \theta_k^M, \theta_k^U\right)$. Where θ_k^L is the lower bound of the efficiency value of $\tilde{\theta}_k$,θ_k^U is the upper bound of the efficiency value of $\tilde{\theta}_k$.

The concept of fuzzy set theory can incorporate the DEA model. In next section, we will explore the concept of the fuzzy set theory and fuzzy regression model.

2.2. Fuzzy Set Theory and Fuzzy Linear Regression

(1) The Fuzzy Number

From the aspect of fuzzy set theory, there are many different types of fuzzy numbers, such as triangular type or multilateral type, which are based on a membership function. The triangular type fuzzy number is enfolded with the features of continuity, convex fuzzy subset and regularity. Triangular fuzzy data has shown the interference rules, which can be expressed in an exact analytical way (Baets and Kerre, 1993). The fuzzy number concept can be used to represent the feature of uncertainty. The triangular fuzzy membership function is defined as Eq.(3).